THEORETICAL ANALYSIS OF THE HEATED JET PUMP AND DESIGN OF A TEST FACILITY

WENDELL CLAYTON RIDDER
and
CARL RICHARDS SUMMERS

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by

Wendell Clayton Ridder
Lieutenant, United States Navy
B.S., University of Kansas, 1960

and

Carl Richards Summers
Lieutenant, United States Navy
B.S., University of Colorado, 1958

[redacted]

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ABSTRACT

The jet pump is a device that compresses low-pressure fluid by means of a high energy jet. This device is simple, reliable and light weight, and is attractive for various uses including eventual application to boundary layer control for aircraft.

In this study the theoretical flow of a perfect gas through constant pressure and constant area jet pumps is predicted by analyzing the equations of continuity, energy and momentum. Of particular interest is the effect of heating the high energy jet. The parameters describing optimum heated jet pumps are determined. The ideal pumps presented herein represent upper performance limits for actual devices.

Also included is a complete description and design of a facility for testing heated jet pumps. Specific test configurations are analyzed, and performance curves are illustrated. These curves facilitate comparison between theory and experiment.

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TABLE OF SYMBOLS

Symbols

A	cross-sectional area
a^*	speed of sound at Mach Number equal unity
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_M	momentum coefficient
C_p	power coefficient
C_{PM}	power-momentum coefficient
C_Q	heat coefficient
C_{QM}	heat-momentum coefficient
D	diameter of duct
g	acceleration given a unit mass by unit force
h	static enthalpy
H	total enthalpy
L	length of duct
$\dot{m}$	mass flow rate
M^*	velocity ratio, V/a^*
p	static pressure
P_t	total pressure
q	heat
R	gas constant
s	entropy
T	absolute temperature
V	velocity

Table of Symbols (Cont'd)

Symbol	
x	mass flow ratio, $\dot{m}_2/\dot{m}_3$
y	area ratio, A_2/A_3
γ	ratio of specific heats, c_p/c_v
Δ	difference or change of
η	efficiency
ρ	density
$\phi(M^*)$	defined velocity function
$\psi(M^*)$	defined velocity function
$(\ )_t$	signifies stagnation state
$(\ )_2$	signifies station location, numbers 1 through 6
$(\ )^*$	signifies the state at which the Mach Number is unity; does not apply to M^*

1. Introduction

A jet pump is a device designed to draw a given mass flow of fluid from a low-pressure source and deliver it to a region of higher pressure by means of a high energy jet of air. Basically, this analysis was undertaken to investigate the theoretical operation of jet pumps, and to recommend a test facility design to provide for the comparison of theoretical and experimental data. The ultimate objective of this investigation is the eventual application of jet pumps to boundary layer control on aircraft.

The jet pump is attractive for an application of this nature due to its reliability, simplicity, and light weight. Also, this device can be readily incorporated into a jet aircraft's power plant cycle. For certain conditions, such as full-throttle operation, it is conceivable that the mechanical power available to the jet pump may be limited. Hence, it is of interest to investigate if the mechanical power required for a given application can be reduced by heating the primary jet air.

The theoretical analysis of ideal jet pump performance contained herein considers the effect of this heating. Constant area and constant pressure mixing processes are investigated. In all cases it is possible to predict "optimum conditions" for specified jet pump configurations. Since all losses except mixing losses are neglected, the results of this study represent a performance limit that actual jet pumps can approach, but never exceed. The final results shown in Appendix C were determined by means of the Control Data Corporation 1604 digital computer. This theoretical analysis is an extension of the work initiated by

Belter in Reference 1.

For any study to be complete it is necessary to substantiate the theoretical results by experimental data. In view of this fact, a design for a heated jet pump test facility is presented. Several test facility configurations along with their material cost estimates are included.

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2. Theoretical analysis

Definition of performance coefficients

A jet pump must be able to produce a required discharge momentum. For this output, a certain amount of mechanical power and heat energy must be supplied to the jet air. Reference 1 expresses the power and heat inputs and discharge momentum as dimensionless coefficients. The arbitrary reference dimensions chosen were for ambient air at sonic velocity and the hypothetical area, A_6, necessary to pass the discharge from the jet pump at sonic velocity.

a. Momentum coefficient

The momentum coefficient is defined as the dimensionless discharge momentum.

$$C_M = \frac{\rho_5 A_5 V_5}{\rho_o^* A_6 a_o^*} \tag{1}$$

b. Power coefficient

The power coefficient is defined as the dimensionless power supplied to the primary air.

$$C_P = \frac{\rho_3 A_3 V_3}{\rho_o^* A_6 a_o^*} \quad \frac{c_p \Delta T_c}{\frac{a_o^{*2}}{2gJ}} \tag{2}$$

where ΔT_c is the temperature change through the compressor.

c. Heat coefficient

The heat coefficient is defined as the dimensionless heat supplied to the primary air.

$$C_Q = \frac{\rho_3 A_3 V_3}{\rho_o^* A_6 a_o^*} \quad \frac{c_p \Delta T_B}{\frac{a_o^{*2}}{2gJ}} \tag{3}$$

where ΔT_B is the temperature change through the burner.

Derived coefficients

It is convenient to compare the mechanical power and heat energy to some significant parameter of the jet pump in order to determine the effectiveness of any design. The reference parameter chosen was the discharge momentum of the jet pump. By dividing the power coefficient and the heat coefficient by the momentum coefficient, it is possible to obtain derived power-momentum and heat-momentum coefficients.

$$C_{PM} = \frac{C_P}{C_M} = \frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5} \frac{a_o^*}{V_5} \frac{c_p \Delta T_c}{\frac{a_o^{*2}}{2gJ}} \tag{4}$$

$$C_{QM} = \frac{C_Q}{C_M} = \frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5} \frac{a_o^*}{V_5} \frac{c_p \Delta T_B}{\frac{a_o^{*2}}{2gJ}} \tag{5}$$

It is shown in Appendix A that C_{PM}, C_{QM}, and C_M can be reduced to the following:

$$C_M = \frac{P_{t5}}{P_{to}} \left\{ \frac{\gamma+1}{\gamma-1} \left[1 - \frac{P_{to}}{P_{t5}} \right]^{\frac{\gamma-1}{\gamma}} \right\}^{1/2} \tag{6}$$

$$\frac{C_{PM}}{\left[\left(\frac{P_{t1}}{P_{to}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} = \frac{C_{QM}}{\left[\frac{T_{t1}}{T_{to}} - \left(\frac{P_{t1}}{P_{to}} \right)^{\frac{\gamma-1}{\gamma}} \right]} = \frac{\left(\frac{\gamma+1}{\gamma-1} \right) \frac{1}{M_5^*}}{\sqrt{(x-1)\left(x + \frac{T_{t1}}{T_{to}} \right)}} \tag{7}$$

where x is the mass flow ratio, $\dot{m}_2/\dot{m}_3$.

Equation (6) shows that C_M depends only on the discharge total pressure ratio, P_{t5}/P_{to}. In particular, C_M is independent of the amount of heat supplied. By specifying the desired output pressure ratio, P_{t5}/P_{to}, the required C_M is determined.

Consider the case where P_{t_1}/P_{t_o} , T_{t_1}/T_{t_o} and P_{t_5}/P_{t_o} are fixed. Equation (7) shows that under these conditions C_{PM} and C_{QM} will be a minimum when the mass flow ratio, χ , is a maximum. It is noted that Equation (7) is invariant for both the constant pressure and constant area cases, although the maximum value of χ may differ in the two cases.

In a sense, C_{PM} is a measure of the compressor power required and C_{QM} is a measure of the heat energy required. Similarly, C_M is a measure of the demanded discharge momentum. Any improvement in design and performance should be reflected by a decrease in power and/or fuel requirements. Therefore, as may be seen from Equation (7), a necessary and sufficient criterion for optimum performance is that the mass flow ratio, χ , be a maximum. Henceforth, an optimum pump is defined as one that has the maximum possible χ for specified values of P_{t_1}/P_{t_o} , T_{t_1}/T_{t_o} , and P_{t_5}/P_{t_o} . It will be shown later that under these conditions an optimum pump corresponds to a specific area ratio, A_2/A_3 . Any other value of area ratio will produce less than the maximum possible mass flow ratio.

Efficiency

Another performance parameter of some interest is the mechanical efficiency which is, of course, based on energy considerations, namely,

$$\eta_M \equiv \frac{\text{Final available energy (Isentropic)}}{\text{Initial available energy (Isentropic)}}$$

$$\eta_M = \frac{\dot{m}_5 \, (H_5 - h_5)}{\dot{m}_3 \, \Delta H_c}$$

It is shown in Appendix A that the above definition is reducible to the result:

$$\eta_M = \frac{\left\{ 1 - \left(\frac{P_{t0}}{P_{t5}}\right)^{\frac{\gamma-1}{\gamma}} \right\}}{\left\{ \left(\frac{P_{t1}}{P_{t0}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} \left(\chi + \frac{T_{t1}}{T_{t0}} \right) \tag{8}$$

From Equation (8), η_M , like the other performance parameters, will be optimum when χ is a maximum for specified conditions of pressure and temperature ratio.

Constant pressure development

This analysis is an optimization of the performance of the constant pressure heated jet pumps which were described in Reference 1. A typical constant pressure mixing jet pump is shown in Figure 1 along with the associated h-s diagram. The basic development of the constant pressure case is from Reference 2. The many details of the development are contained in Appendix A.

The usual isentropic relationships were used to solve for conditions in the primary (jet) system and in the secondary system up to the mixing tube entrance. It is also assumed that the specific heats remain constant.

The three basic equations that have to be satisfied for constant pressure mixing ($p_2 = p_4$) are continuity, energy, and momentum.

Continuity:

$$\dot{m}_2 + m_3 = \dot{m}_4 \tag{9}$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \tag{10}$$

Momentum:

$$(\dot{m}_3 + \dot{m}_2) V_4 - (\dot{m}_3 V_3 + \dot{m}_2 V_2) = (p_3 - p_2) A_3 \tag{11}$$

It is noted that the resultant pressure forces in Equation (11) are zero when the velocity in the primary nozzle is subsonic.

The equations of continuity, energy, and momentum can be expressed in non-dimensional terms. This development is shown in Appendix A.

Continuity:

$$\frac{A_4}{A_3} = \sqrt{(1+\chi)\left(\frac{T_{t1}}{T_{to}} + \chi\right)} \left\{ \frac{\frac{P_{t1}}{P_{to}}}{\left(\frac{P_{t5}}{P_{to}}\right)\sqrt{\frac{T_{t1}}{T_{to}}}} \right\} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*} \tag{12}$$

Energy:

$$\frac{T_{t1}}{T_{to}} + \chi = (1+\chi)\frac{T_{t4}}{T_{to}} \tag{13}$$

Momentum:

$$\sqrt{\frac{T_{t1}}{T_{to}}}\, M_3^* + \chi M_2^* + \sqrt{\frac{T_{t1}}{T_{to}}}\, \xi = (1+\chi)\sqrt{\frac{T_{t4}}{T_{to}}}\, M_4^* \tag{14}$$

where,

$$\xi = \frac{1}{\gamma}\left\{ 1 - \left(\frac{P_{to}}{P_{t1}}\right)\left[\frac{1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}}{\frac{2}{\gamma+1}}\right]^{\frac{\gamma}{\gamma-1}} \right\}$$

By fixing P_{t1}/P_{to} , T_{t1}/T_{to} , P_{t5}/P_{to} , and M_2^* , it is possible to obtain an explicit solution from Equations (12), (13), and (14) since there remains only three unknowns χ , T_{t4}/T_{to} , and A_4/A_3 .

Equations (13) and (14) are solved simultaneously for χ and T_{t4}/T_{to} . By eliminating T_{t4}/T_{to} between the two equations, the following quadratic equation in χ is obtained.

$$(M_4^{*2} - M_2^{*2})\chi^2 + \left\{\left(\frac{T_{t1}}{T_{to}} + 1\right)M_4^{*2} - 2\sqrt{\frac{T_{t1}}{T_{to}}}\, M_2^{*2}\,(M_3^* + \xi)\right\}\chi + \frac{T_{t1}}{T_{to}}\left\{M_4^{*2} - (M_3^* + \xi)^2\right\} = 0 \tag{15}$$

Equation (15) can be solved using the quadratic formula

$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

$$a = M_4^{*2} - M_2^{*2}$$

$$b = \left(\frac{T_{t1}}{T_{to}} + 1\right) M_4^{*2} - 2\sqrt{\frac{T_{t1}}{T_{to}}}\, M_2^* \left(M_3^* + \xi\right)$$

$$c = \frac{T_{t1}}{T_{to}} \left\{ M_4^{*2} - \left(M_3^* + \xi\right)^2 \right\}$$

Since the mass flow ratio, χ , must be positive and real, one limiting condition is that $b^2 - 4ac \geq 0$. For all cases investigated this condition is satisfied. Other limiting conditions are $M_2^* \leq 1.0$ and $M_3^* \leq 1.0$ since the primary and secondary nozzles are only converging, not converging-diverging. Both roots of Equation (15) were determined. One of these was negative and, therefore, was discarded as meaningless.

From the solution of Equation (15) it was possible to determine T_{t4}/T_{to} from the energy Equation (13) and A_4/A_3 from the continuity Equation (12). A_2/A_3 can be solved directly from the mass flow ratio.

The computer was used to obtain solutions to Equations (12), (13), and (14). The parameters P_{t1}/P_{to} , T_{t1}/T_{to} , P_{t5}/P_{to} , and M_2^* were varied systematically to obtain families of constant pressure pumps. The optimum pump from each family was determined. Appendix C contains solutions for each optimum pump for the conditions specified.

Constant Area Development

This analysis is an investigation and optimization of the ideal flow of a perfect gas through a constant area jet pump. Figure 2 shows a sketch and a general h-s diagram of a constant area pump. This figure indicates the notation to be used to denote station locations throughout this development.

Much as before, this analysis involves the solution of the continuity, energy, and momentum equations. These equations may be written in absolute form as follows:

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \tag{16}$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \tag{17}$$

Momentum:

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \tag{18}$$

It is shown in Appendix A that the above equations can be non-dimensionalized to yield the following relationships:

Continuity:

$$y\phi(M_2^*) + \frac{P_{t1}}{P_{to}}\sqrt{\frac{T_{to}}{T_{t1}}}\,\phi(M_3^*) = (1+y)\frac{P_{t4}}{P_{to}}\sqrt{\frac{T_{to}}{T_{t4}}}\,\phi(M_4^*) \tag{19}$$

Energy:

$$y\phi(M_2^*) + \frac{P_{t1}}{P_{to}}\sqrt{\frac{T_{t1}}{T_{to}}}\,\phi(M_3^*) = (1+y)\frac{P_{t4}}{P_{to}}\sqrt{\frac{T_{to}}{T_{t4}}}\,\phi(M_4^*) \tag{20}$$

Momentum:

$$y\psi(M_2^*) + \frac{P_{t1}}{P_{to}}\,\psi(M_3^*) = (1+y)\frac{P_{t4}}{P_{to}}\,\psi(M_4^*) \tag{21}$$

where:

$$\phi(M^*) = \left(1 - \frac{\gamma-1}{\gamma+1}M^{*2}\right)^{\frac{1}{\gamma-1}} M^*$$

$$\psi(M^*) = \left(1 - \frac{\gamma-1}{\gamma+1}M^{*2}\right)^{\frac{1}{\gamma-1}}(1 + M^{*2})$$

and y is the area ratio, A_2/A_3 .

The development of Equations (19), (20) and (21) is from Reference 2. A similar development is included in Reference 1. However, an error

was found in Equation (37) of Reference 1 rendering incorrect solutions for the constant area case presented therein.

It is assumed that the parameters P_{t1}/P_{to} , T_{t1}/T_{to} , and P_{t5}/P_{to} are given. Values of the secondary velocity ratio, M_2^* , are systematically chosen. Once M_2^* is fixed the primary velocity ratio, M_3^* , can be determined as shown in Appendix A. The remainder of the problem resolves into a solution of Equations (19), (20) and (21) for the unknowns M_4^* , y and T_{t4}/T_{to} ; and satisfying the boundary condition that the value of P_{t4}/P_{to} so obtained must equal the value originally specified for P_{t5}/P_{to} .

Since the resulting relationships are transcendental, it is necessary to solve the equations by iterative procedures. It is thereby possible to achieve solutions for numerous families of constant area heated jet pumps.

The routine employed to solve the equations is to choose incremental values for M_2^* . Choosing M_2^* fixes all the known parameters not previously specified. A trial value of the area ratio, y, is assumed. The mass flow ratio is then solved by the relationship:

$$\chi = \dot{m}_2/\dot{m}_3 = y \frac{P_{to}}{P_{t1}} \sqrt{\frac{T_{t1}}{T_{to}}} \; \frac{\phi(M_2^*)}{\phi(M_3^*)} \tag{22}$$

Now it is possible to solve for the discharge total temperature ratio, T_{t4}/T_{to} , by using the equation:

$$\frac{T_{t4}}{T_{to}} = \frac{\chi + T_{t1}/T_{to}}{\chi + 1} \tag{23}$$

This procedure determines all the parameters except the exit velocity ratio, M_4^* . The final equation solved is a quadratic in terms of M_4^* .

$$\frac{M_4^*}{1+M_4^{*2}} = \sqrt{\frac{T_{t0}}{T_{t4}}}\ \frac{y\,\phi(M_2^*) + (P_{t1}/P_{t0})\sqrt{\frac{T_{t1}}{T_{t0}}}\,\phi(M_3^*)}{y\,\psi(M_2^*) + (P_{t1}/P_{t0})\,\psi(M_3^*)} \tag{24}$$

The above equation gives a supersonic and a subsonic solution for M_4^*. For this particular analysis (i.e., M_2^* and $M_3^* \leq 1$ and constant area mixing), the supersonic solution for M_4^* is meaningless. This means to say that M_4^* cannot be supersonic for a constant area pump when the inlet velocity ratios, M_2^* and M_3^*, are limited to one or less. However, the supersonic solution of M_4^* may be of interest if M_3^* and/or M_2^* are greater than one.

The preceding procedure gives a solution to the equations; however, it is also necessary to satisfy the boundary conditions. The resulting exit total pressure ratio, P_{t4}/P_{t0}, is checked against the desired discharge total pressure ratio, P_{t5}/P_{t0}. If these quantities are not equal, the area ratio is adjusted in order to achieve the necessary conditions at the end of the mixing section. This process is repeated to determine families of constant area jet pumps.

Figure 3 shows the performance curves for two families of constant area pumps. As defined previously, the optimum pumps are those which achieve a maximum mass flow ratio for given values of P_{t1}/P_{t0}, T_{t1}/T_{t0}, and P_{t5}/P_{t0}. It is observed that optimum conditions are reached in two different ways for the constant area case.

Again referring to Figure 3, curve A passes through a true mathematical maximum in the sense that $\frac{dM_2^*}{d\chi} = 0$. This curve shows that as M_2^* increases the mass flow ratio increases up to a maximum and then decreases. This situation will be referred to as case A hereafter.

Curve B shows the other limiting case of the constant area system.

This curve increases up to a point of maximum mass flow ratio and simply ceases. This phenomena is not clearly understood. Constant area pumps that reach optimum conditions in this manner will be known as case B.

The computer solutions in Appendix C contain a complete set of optimum constant area heated jet pumps for the conditions specified. The non-starred optimum pumps correspond to case A, and the starred pumps are limited in a manner illustrated by case B. These results will be discussed in a following section.

Discussion of Theoretical Results

One significant indication of the performance of a jet pump is the amount of energy that must be supplied to produce the discharge momentum demanded. The coefficients C_{PM} and C_{QM} are simply numbers indicating the energy supplied; and the coefficient C_M is a measure of the discharge momentum required. These coefficients show the effect of changing the parameters P_{t1}/P_{to}, T_{t1}/T_{to}, and P_{t5}/P_{to}.

Figures 4 through 9 are graphs of C_{PM} compared to C_{QM} for families of optimum constant pressure and constant area jet pumps. These graphs are basically maps formed by curves of constant supply pressure and temperature ratios. One of the most significant results illustrated by these maps is the trade-off between the heat energy and compressor power necessary to produce the desired C_M.

For example, consider Figure 4. If the supply pressure ratio is fixed, C_{PM} decreases as the supply temperature ratio increases. This indicates that <u>the compressor power required is reduced by heating the primary air</u>. Conversely, for a constant temperature ratio, C_{QM} is

reduced as P_{t1}/P_{t0} is increased. For the highest temperature ratio and the lowest pressure ratio, the compressor power is minimal. It is also observed that the greatest incremental decrease in C_{PM} occurs between temperature ratios of 1 and 2 along lines of constant pressure.

Now consider the sequence of Figures 4 through 6. It can be seen that as the momentum coefficient demanded is increased, the maps tend to move up and to the right. This, of course, corresponds to higher supply energy requirements. Also, it is noted that the effect of heating is greater as the demand on the pump is increased.

Figures 10 through 15 are graphs of maximum mass flow ratio versus M_2^* . These maps illustrate the performance limitations of the pumps. It must be understood that each point on a map represents a different pump. Therefore, any one curve provides an array of ideal design possibilities. This should not be confused with the performance variation due to changing operating conditions of an individual pump.

Each map shows that as T_{t1}/T_{t0} is increased along lines of constant P_{t1}/P_{t0} , the maximum mass flow ratio increases. This increase is greatest up to a temperature ratio of 2. Beyond this temperature ratio, the effect of heating is less apparent.

Figures 13 through 15 are performance maps for optimum constant area pumps. The lines of constant temperature ratio are continuous; however, there are points of discontinuity in the slope of each line. The dashed curve connecting these points divides each map into two distinct regions. The lower left area of the map corresponds to a flow limitation as shown by curve A of Figure 3. The area to the right of the dashed curve exhibits a flow cut-off similar to that illustrated by case B. These figures also show that the secondary mass flow rate

($\dot{m}_2$) decreases as the discharge total pressure ratio increases.

Appendix C contains the solutions of the computer programs for the constant area and constant pressure cases. Each line of data represents the parameters defining one optimum pump. The momentum coefficient is constant for each page of data. Area, velocity, and temperature ratios were included in these results to provide additional criteria for jet pump design. The results show that for a given C_M and T_{t1}/T_{to} , the discharge total temperature ratio can be decreased by increasing P_{t1}/P_{to} . This fact could be an important consideration in choosing the material for the mixing tube section.

The important effect of heating the primary air to decrease the compressor power required has been shown. It must be realized that these are idealized cases that represent the maximum performance attainable. Actual jet pumps can approach but never exceed these limits.

Fixed Area-Variable Discharge Pressure

The final theoretical analysis considered is to investigate the operating characteristics of jet pumps with a fixed area ratio (A_2/A_3). By systematically varying the supply pressure and temperature ratios, it is possible to predict the envelope of operation for a given pump configuration. These predicted values are again based on ideal conditions; therefore, they represent the maximum performance attainable.

The fundamental objectives of this analysis are; first, to marry the previously discussed theory to an actual hardware design; second, to determine the design limiting parameters such as size, available compressor power and material limitations; and finally, to predict the performance curves of the jet pumps designed.

The theory of the ideal flow through a fixed area pump is similar to the analysis discussed in the constant area section. However, by fixing the area ratio (A_2/A_3) and allowing P_{ts}/P_{to} to vary it is possible to obtain explicit solutions for the equations of continuity, energy and momentum. Appendix C contains an example of the data obtained from the computer program used to predict each pump's performance.

The constant area system is selected primarily due to its relatively simple mixing section design. The supply temperature ratios are limited to three or less, and supply pressures are investigated up to ratios of 2.4. Mixing tube diameters of four, five, and six inches are chosen. This choice is predicated on the nominal pipe sizes available. The primary jet nozzles are designed to give a good spread of area ratios (A_2/A_3). The final result is nine different jet pump configurations. The table below shows the area ratios of the jet pumps investigated.

		NOZZLE EXIT DIAMETER		
		2.00"	2.25"	2.60"
MIXING TUBE DIAMETER	4"	3.000	2.160	1.367
	5"	5.250	3.938	2.698
	6"	8.000	6.111	4.325

Table 1. Area Ratios Selected For Proposed Design

Figures 16 through 24 show the predicted operating maps for the nine

fixed area pumps considered. These curves are prepared primarily for a quick comparison of theoretical and experimental results. Consider the pump with an area ratio of 2.16, shown on Figure 16. Assume that $T_{t1}/T_{to} = 1.0$, $P_{t1}/P_{to} = 1.6$ and the back pressure ratio (P_{t5}/P_{to}) is adjusted to 1.2. Entering the map with these values predicts a mass flow ratio of 1.07. Now if the primary fluid is heated to $T_{t1}/T_{to} = 3.0$, it can be seen that the mass flow ratio increases to 1.33. This is another example of how heating favorably influences the performance of a jet pump.

It must be clearly understood that these mass flow ratios are simply upper performance limits. Actual jet pumps will not reach these values due to friction and secondary mixing effects.

Theoretical conclusions

For fixed input and output conditions, all performance parameters vary favorably with an increasing mass flow ratio.

For fixed supply and demand conditions, there is one and only one area ratio, A_2/A_3, that will produce the maximum possible mass flow ratio and hence the best all around performance.

Heating the primary air decreases the compressor power required, i.e., there is a trade-off between heat and power.

The optimum pumps are idealized cases that represent the maximum performance attainable. Actual jet pumps can approach but never exceed these limits.

The ordinary ideas of efficiency are not necessarily decisive in the application of this device to boundary layer control.

3. Jet Pump Test Facility Design

The test facility proposed for studying heated jet pump operation is shown in Figure 25. Basically, it consists of (1) a primary flow system for supplying jet air at a desired pressure and temperature; (2) a secondary air supply system; (3) a set of interchangeable converging jet nozzles; and (4) the constant area mixing tubes.

The test apparatus was designed to be a fixed installation exhausted to the atmosphere. The primary purpose of this facility is to provide fundamental information on jet pump design parameters, and to establish correlation between experimental and theoretical data.

Primary Air Supply System

The primary flow section includes the compressed air supply and fuel supply systems and the combustion chamber assembly. The purpose of the primary system is to provide heated compressed air to the jet pump.

The flow of compressed air to the jet pump is controlled by means of an electrically actuated butterfly valve. It may assume any position between fully closed and fully opened. Thus, this valve controls the pressure of the primary fluid downstream.

After passing through the throttle valve, the compressed air flows into the primary system plenum chamber. The dimensions of this reservoir are arbitrary; however, the chamber must be large enough to serve as a settling tank.

The compressed air leaves the plenum chamber and passes through a square-edged orifice installed in the four inch mild steel exit pipe.

This orifice measures the flow rate of the primary air, $\dot{m}_3$. The compressed air is then directed into the burner assembly.

The burner unit is a small cannular combustion chamber obtained from an auxiliary gas turbine compressor used for starting jet aircraft engines. The fuel nozzle, igniter plugs and the flame tube are integral parts of the burner unit. The operating range imposed on the jet pump (i.e., $P_{t1}/P_{to} \leq 2.4$ and $T_{t1}/T_{to} \leq 3.0$) is well within the capabilities of the combustion chamber.

Fuel is supplied to the burner from a pressurized fuel tank. The fuel pressure is maintained constant by means of a pressurized bottle of inert gas. Fuel flow is manually controlled by a needle valve installed in the fuel supply line. This valve sets the fuel/air ratio and is the primary means of controlling the total supply temperature, T_{t1} . The ignition system consists simply of two 12 volt storage batteries connected in series and an ignition control switch.

After the primary air is heated, it passes through a 17 inch section of 5 inch stainless steel pipe. This pipe section is inserted into the secondary air plenum chamber and provides a male fitting for the primary jet nozzle. Total supply temperature and pressure are measured by means of a fixed Kiel-temperature probe. These two values combined with the flow rate are the parameters of prime importance in the primary supply system.

The primary air then passes through a converging nozzle and is ejected into the mixing tube section. The three interchangeable nozzles designed are shown on Figures 26 through 28. These stainless steel nozzles must be machined with close dimensional control and polished to

a mirror finish. They are tapered to ensure uniform flow of primary and secondary air at the mixing tube entrance. A lubricant, such as "silver goop," should be applied to the nozzle threads prior to installation. This material prevents "welding" at elevated temperatures.

Secondary Air System

The secondary air system simply provides a means to measure the secondary air flow rate, $\dot{m}_2$. Basically, it consists of an airtight plenum chamber and the associated piping and fittings for a mass flow determination.

Figure 29 shows the proposed secondary air plenum chamber. The reservoir is constructed of 12 gage sheet steel. Standard flanges are used for all piping connected to the chamber. Care must be exercised during fabrication to ensure the airtight integrity of the plenum. Any leaks in the plenum will render the secondary air mass flow measurement inaccurate.

The alignment of the primary air supply duct and the mixing tube section is important. In order to achieve the desired results it is necessary that the jet nozzle and mixing tube centerlines coincide.

Mixing tube

The length of the mixing tube must be such that optimum mixing is accomplished. Three tubes are available with diameters of four, five, and six inches. Each tube consists of four, 24 inch pipe sections which provide for length to diameter (L/D) ratios of 4.8 to 24. L is the length of the mixing tube downstream of the nozzle exit.

The maximum wall temperature attainable is 1100°F; therefore, a heat resistant alloy is advisable. Type 304, schedule 40 stainless

steel which has a melting range of 2550-2650°F and a scaling temperature of 1550°F was selected as a suitable material. With careful attention to the discharge total pressure for fixed supply pressure and temperature ratios, it would be possible to hold the temperature in the mixing tube low enough so that a mild steel or aluminum could be used. This is considered to be an additional experimental burden that can be alleviated by using stainless steel throughout.

One of the simplifying assumptions was to neglect wall friction; therefore, the internal surface of the mixing tube is to be polished to a smooth surface to minimize frictional losses. Consequently, there should be closer agreement between theory and experiment.

By taking temperature and pressure readings along the mixing tube, it is possible to determine where optimum mixing is achieved. Any additional length will only cause a net additional loss in head due to friction. Many of the design questions concerning optimum length of the mixing section can be answered only by experimentation and by investigation of the kinetics of the mixing process in which all effects are considered.

As shown in Figure 27 an oversized flange connects the mixing tube to the secondary air plenum chamber. Attached to this flange is a bell-mouth annulus machined from hot rolled steel. The converging secondary nozzle is formed by this annulus and the exterior of the primary jet nozzle.

The three mixing tubes can be interchanged and may be used with each of the primary nozzles. This flexibility provides for experimental tests of nine different jet pump configurations.

In an actual application, a diffuser would, of course, be used at

the end of the mixing tube to bring the mixed fluids gradually to atmospheric pressure. For the purposes of the present research, separate diffusers were not designed due to the many varying operating conditions and physical configurations.

The mixing tube is supported by three stanchions similar to the one illustrated in Figure 30. Each stanchion has two turnbuckles which allow alignment in the vertical and horizontal planes.

The handling cart shown in Figure 31 will be used whenever it is necessary to change the mixing tubes. Two carts of this design would be desirable to provide the versatility prescribed for the test facility. The carts are made almost exclusively of aluminum channel and angle stock. The maximum design load is 1000 lbs.

Instrumentation

A typical 24 inch mixing section with the associated instrumentation is shown in Figure 32. Each section contains six static pressure taps. These taps are located at four inch intervals along the entire mixing tube. Since the assumption is made that a constant static pressure exists across each cross section, the static pressure is measured only at the mixing tube walls.

Total pressure and total temperature readings are to be taken with a Kiel-temperature probe. These readings can be taken at any of eight different stations along the mixing tube. At each station there are two probe fittings which permit transverse surveys of temperature and pressure along either of two diameters.

The mounting plates shown at each station support the receiver containing the Kiel-temperature probe. The receiver is mechanically

controlled remotely by a transmitter. This transmitter controls the transverse and angular movement of the probe.

When the probe is inserted in one of the probe fittings, the remaining holes are plugged. These plugs are machined to be flush with the inside walls of the mixing tube.

A butterfly valve is installed in a pipe section after the mixing tube to regulate the discharge total pressure. A second Kiel-temperature probe is available to be installed at the end of the mixing tube to measure this pressure.

This test facility provides the means to determine the primary and secondary mass flow rates. In addition, velocity and temperature profiles may be determined along the length of the mixing tube, thereby enabling determination of an optimum mixing length.

Mass flow measurement

It has been shown that one of the most important performance parameters is the mass flow ratio, $\dot{m}_2/\dot{m}_3$. Figure 33 shows the installation of two square-edged orifices. The orifice plates for the primary and ambient air supply have holes of 2.5 and 3.75 inch diameter respectively. They are installed in slip-on, butt welded flanges which have pressure taps meeting ASME specifications. The two flange taps are connected to a water filled manometer board where the pressure difference , h_w, is measured. The gage pressure P_1 , and temperature t_1 , must be measured ahead of each orifice.

ASME requirements prescribe a minimum straight pipe length ahead of and behind an orifice for a flow rate measurement. The straight pipe length shown in Figure 25 exceeds the minimum requirements; therefore, it is not necessary to install flow straightners.

Reference 3 contains all necessary equations for determination of the flow rates.

Cost estimate

The cost estimate is based on the following assumptions:

a. No old material will be used except the burner and fuel supply system, pressure and temperature gages, and manometer boards.

b. Heat resistant alloy is necessary in the mixing tube.

c. All machining and fabrication can be accomplished by shop personnel.

d. Three nozzles and three mixing tubes are necessary to check a range of configurations.

Two cost estimates are provided:

a. Total material cost with remote electrical control:	\$5555.42
b. Total material cost with remote mechanical control:	\$4787.42

The total man-hour labor estimate is 362 hours.

The material cost could be reduced considerably if only one nozzle and one mixing tube were used. This is shown in the following estimate:

a. Total material cost with remote electrical control:	\$3734.08
b. Total material cost with remote mechanical control:	\$2969.08

A minimum cost estimate is provided based on the following assumptions:

a. One nozzle and one mixing tube will be used.

b. The mixing tube consists of two, 24 inch mixing sections

instead of four.

c. Handling carts are not required.

d. The supply temperature ratio is reduced to $T_{t1}/T_{t_0} = 2.0$. Therefore, stainless steel could be replaced by mild steel.

e. Manual controls are used instead of electrical.

f. As much old material as possible will be used such as plastic tubing, wall taps, copper tubing, connectors, pressure plugs, burner and fuel supply system pressure and temperature gages, and manometer boards.

g. Four inch pipe will be used in the mixing tube.

The total minimum material cost is $1601.50.

An itemized cost estimate is contained in Appendix B.

4. Recommendations for further study

At this stage of the investigation, the most essential requirement is to build the test facility described in this report. If this is not feasible at the present time for economic or other reasons, a further theoretical analysis would be useful to study the effects of wall friction and incomplete mixing. These analytical studies should also include the effects of variable specific heats.

A further recommendation is to proceed with the preliminary design of an experimental boundary layer control system utilizing such a heated jet pump.

It also would be of interest to investigate the performance of a heated jet pump with a converging-diverging primary nozzle.

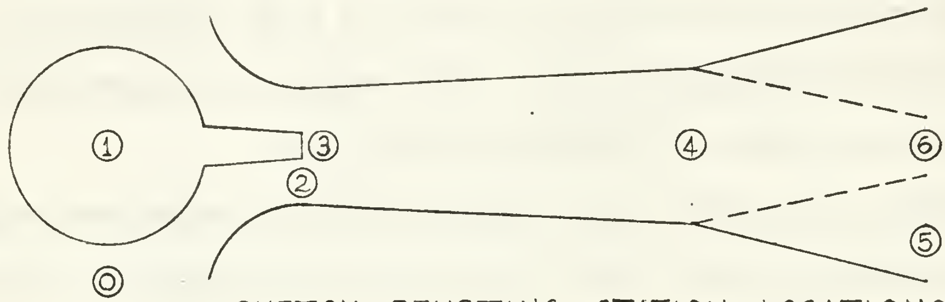
RIDDER
SUMMERS

APRIL 1966

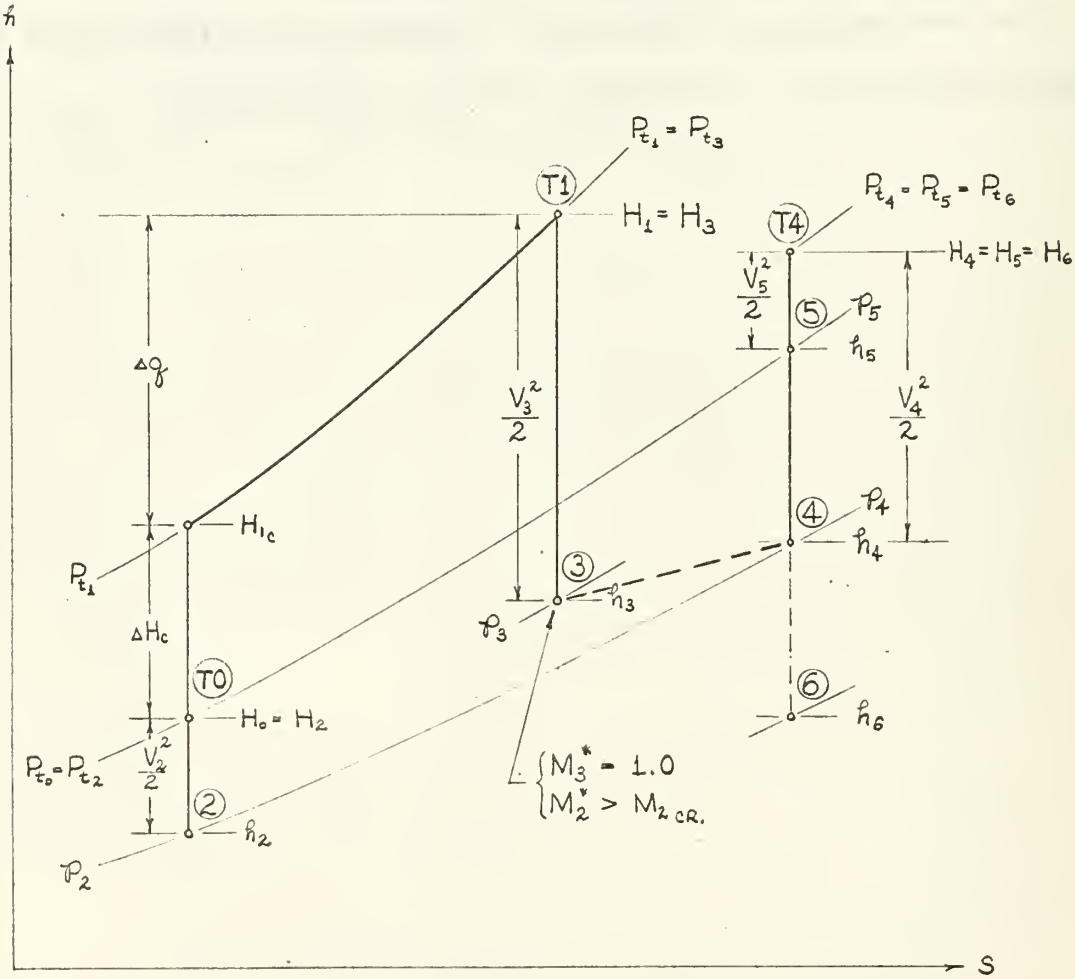
USNPGS DEPARTMENT OF AERONAUTICS

FIGURE 1

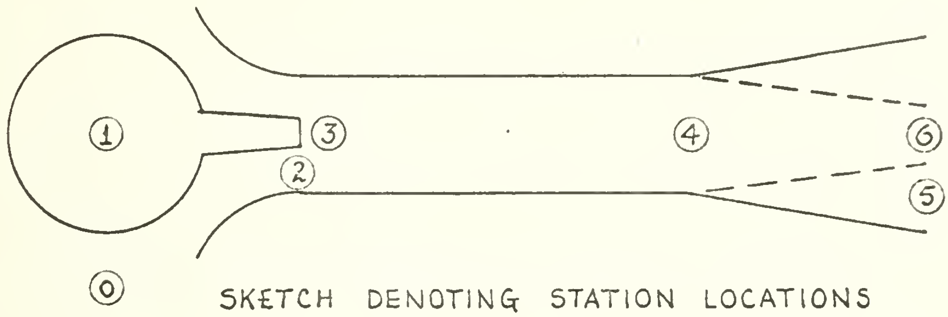
TYPICAL ENTHALPY ENTROPY DIAGRAM FOR A CONSTANT PRESSURE HEATED JET PUMP



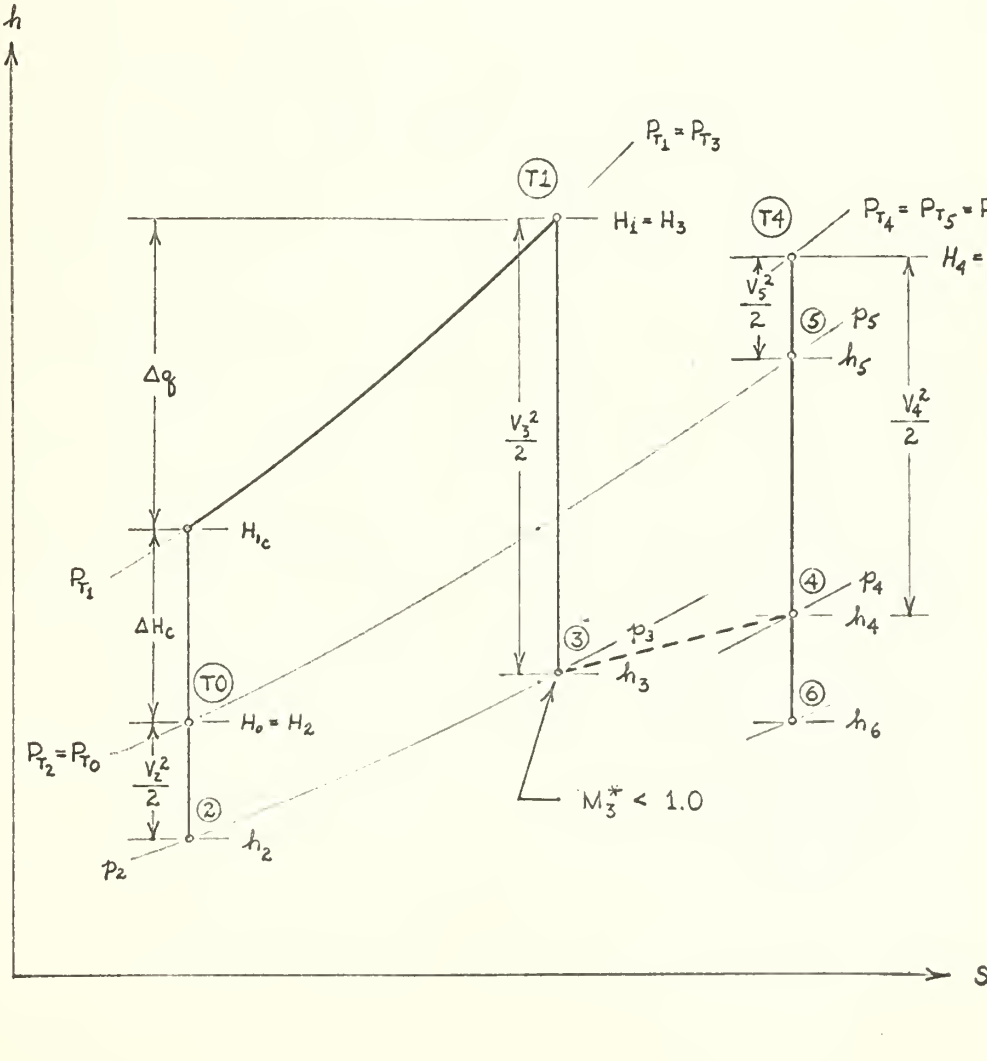
SKETCH DENOTING STATION LOCATIONS



RIDDER SUMMERS	USNPGS DEPARTMENT OF AERONAUTICS	FIGURE 2
	TYPICAL ENTHALPY-ENTROPY DIAGRAM FOR A CONSTANT AREA HEATED JET PUMP	
APRIL 1966		



SKETCH DENOTING STATION LOCATIONS



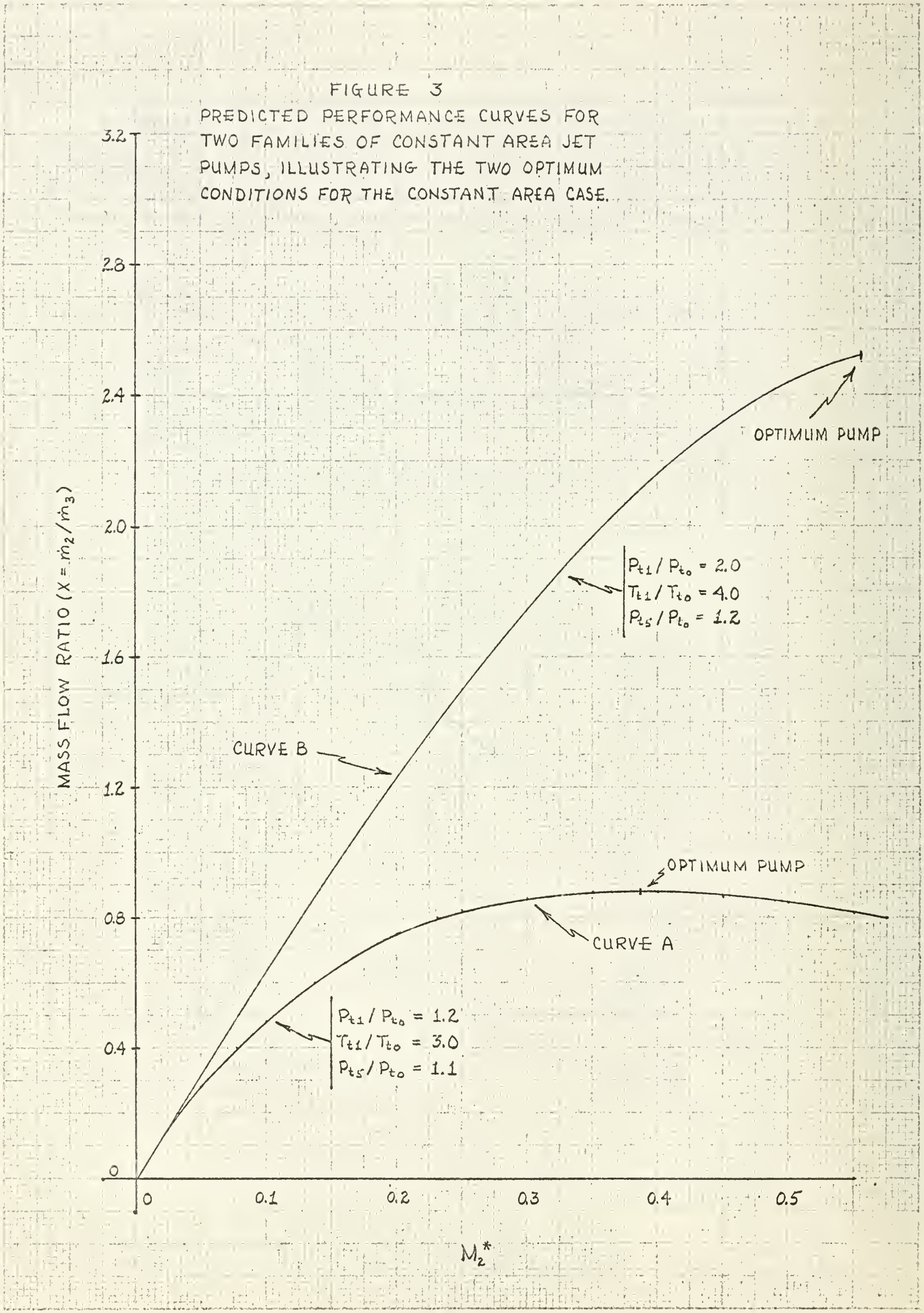


FIGURE 3
PREDICTED PERFORMANCE CURVES FOR TWO FAMILIES OF CONSTANT AREA JET PUMPS, ILLUSTRATING THE TWO OPTIMUM CONDITIONS FOR THE CONSTANT AREA CASE.
MASS FLOW RATIO ($X = \dot{m}_2/\dot{m}_3$)
M_2^*
0
0.1
0.2
0.3
0.4
0.5
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
OPTIMUM PUMP
$P_{t1}/P_{to} = 2.0$
$T_{t1}/T_{to} = 4.0$
$P_{t5}/P_{to} = 1.2$
CURVE B
OPTIMUM PUMP
CURVE A
$P_{t1}/P_{to} = 1.2$
$T_{t1}/T_{to} = 3.0$
$P_{t5}/P_{to} = 1.1$

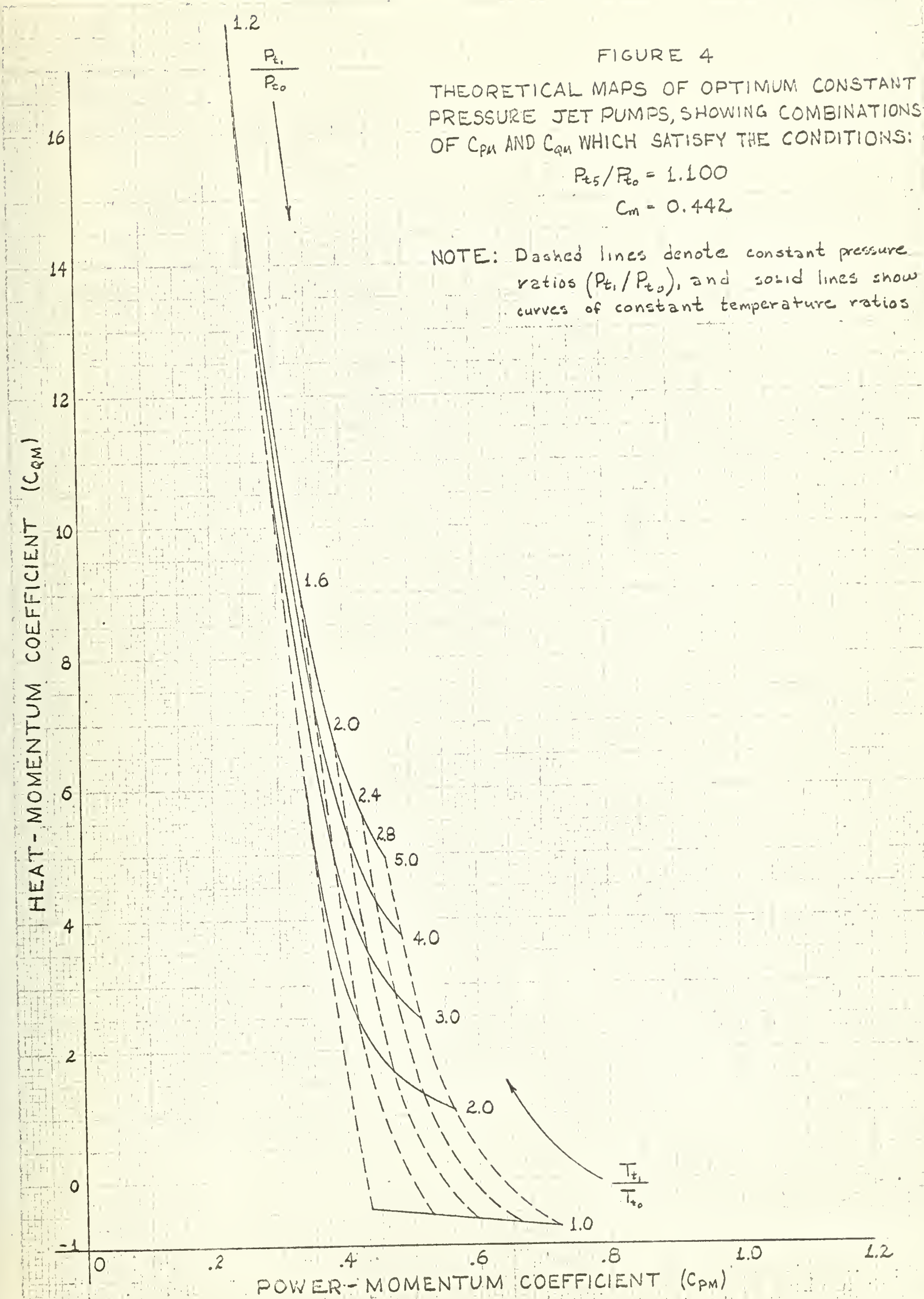


FIGURE 4
THEORETICAL MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:
$P_{t5}/P_{t0} = 1.100$
$C_m = 0.442$
NOTE: Dashed lines denote constant pressure ratios (P_{t1}/P_{t0}), and solid lines show curves of constant temperature ratios
$\frac{P_{t1}}{P_{t0}}$
1.2
1.6
2.0
2.4
2.8
5.0
4.0
3.0
2.0
1.0
$\frac{T_{t1}}{T_{t0}}$
HEAT - MOMENTUM COEFFICIENT (C_{QM})
POWER - MOMENTUM COEFFICIENT (C_{PM})
16
14
12
10
8
6
4
2
0
-1
0
.2
.4
.6
.8
1.0
1.2

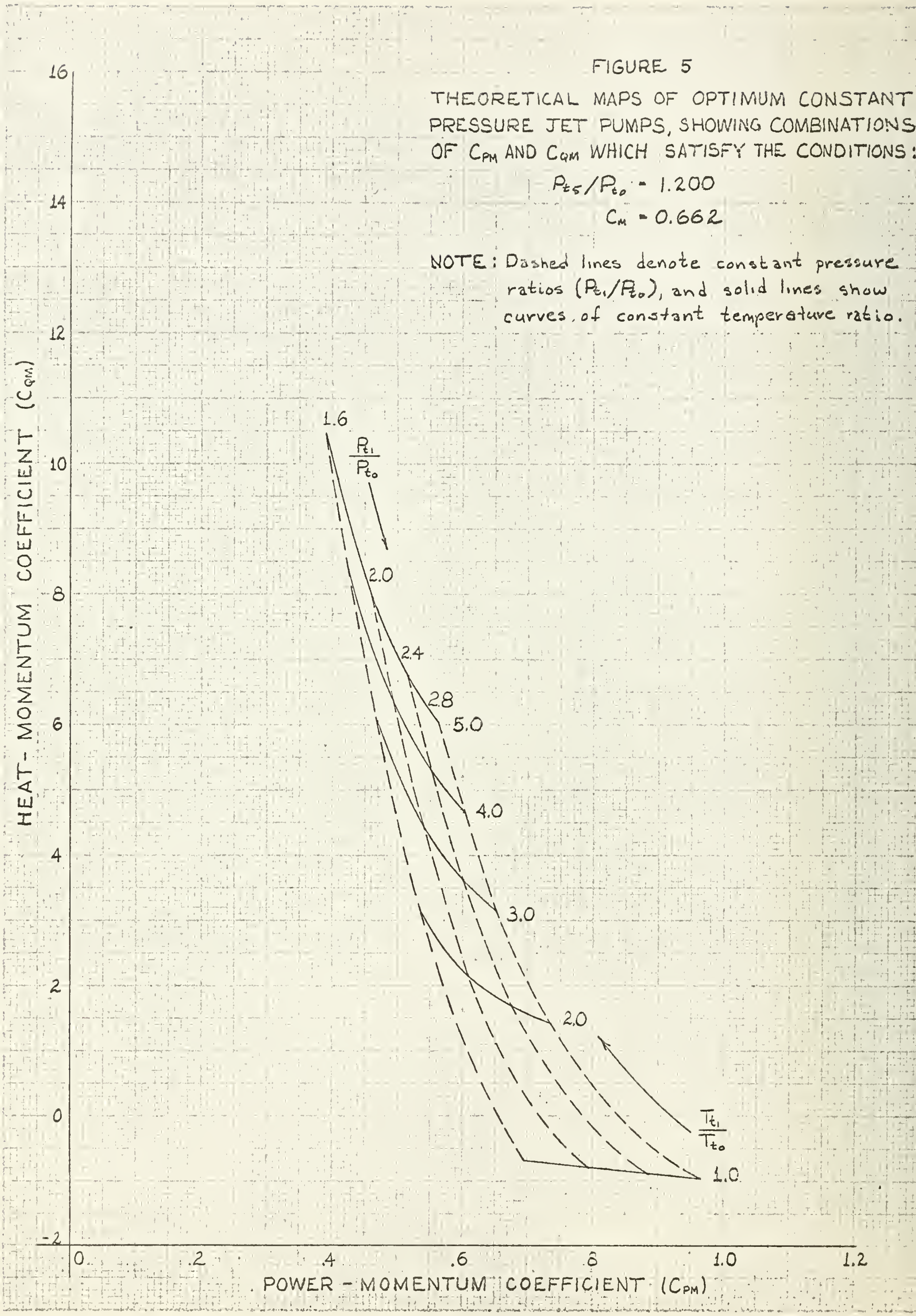


FIGURE 5

THEORETICAL MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:

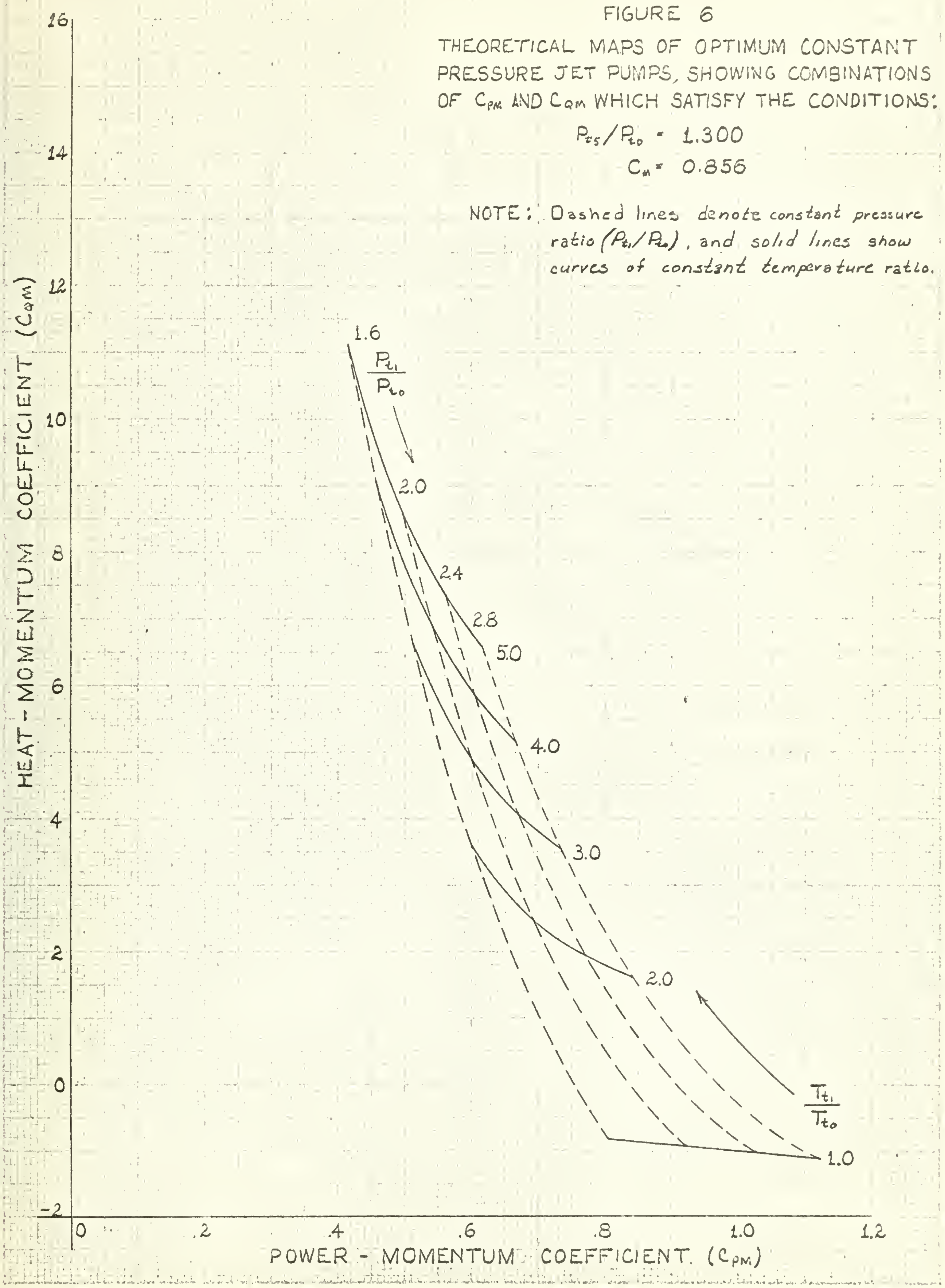


FIGURE 6

THEORETICAL MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:

$P_{t5}/P_{t0} = 1.300$

$C_M = 0.856$

NOTE: Dashed lines denote constant pressure ratio (P_{t_1}/P_{t_0}), and solid lines show curves of constant temperature ratio.

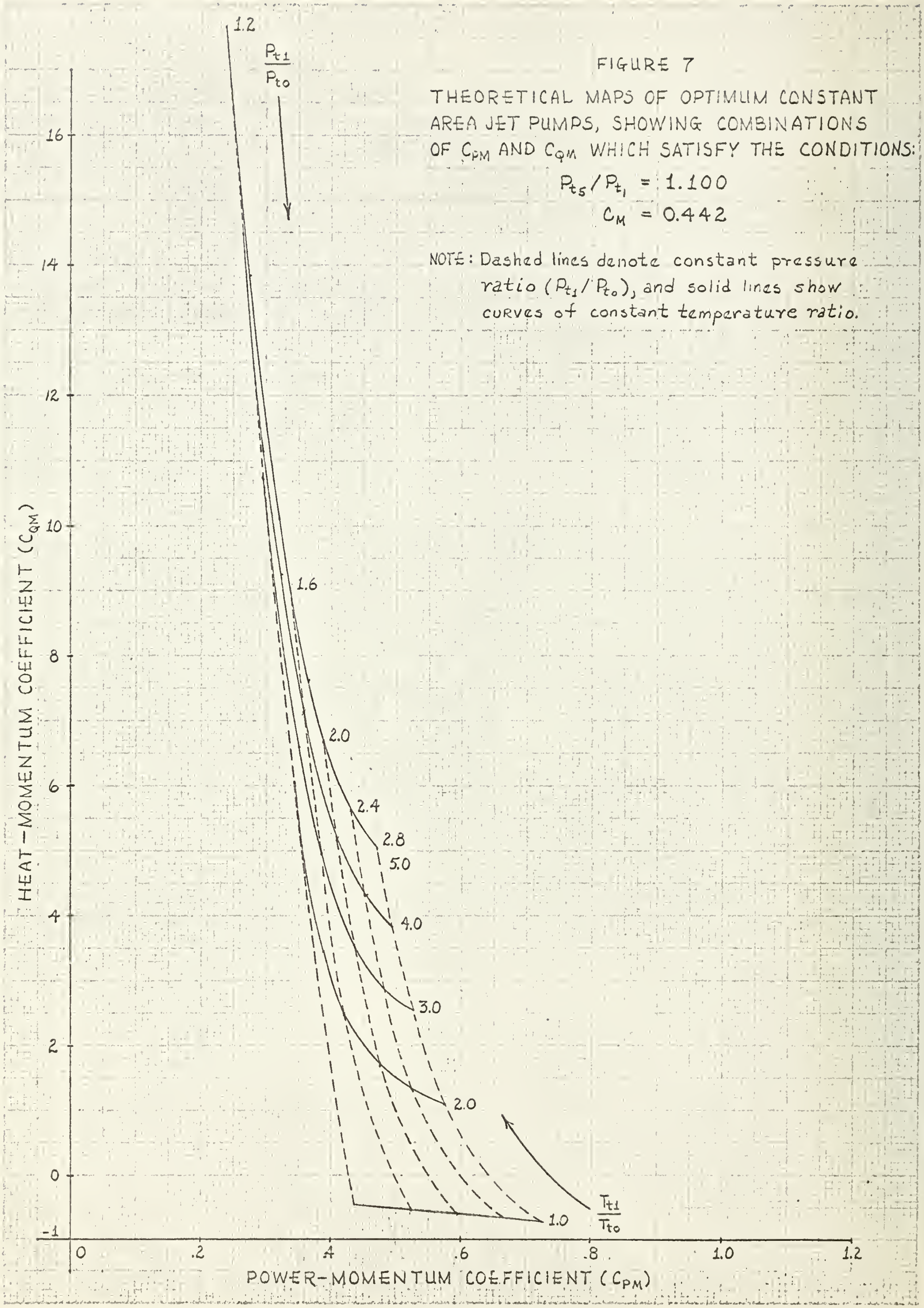


FIGURE 7
THEORETICAL MAPS OF OPTIMUM CONSTANT AREA JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:
$P_{t5}/P_{t1} = 1.100$
$C_M = 0.442$
NOTE: Dashed lines denote constant pressure ratio (P_{t1}/P_{t0}), and solid lines show curves of constant temperature ratio.
$\frac{P_{t1}}{P_{t0}}$
1.2
1.6
2.0
2.4
2.8
5.0
4.0
3.0
2.0
1.0
$\frac{T_{t1}}{T_{t0}}$
HEAT-MOMENTUM COEFFICIENT (C_{QM})
POWER-MOMENTUM COEFFICIENT (C_{PM})
16
14
12
10
8
6
4
2
0
-1
0
.2
.4
.6
.8
1.0
1.2

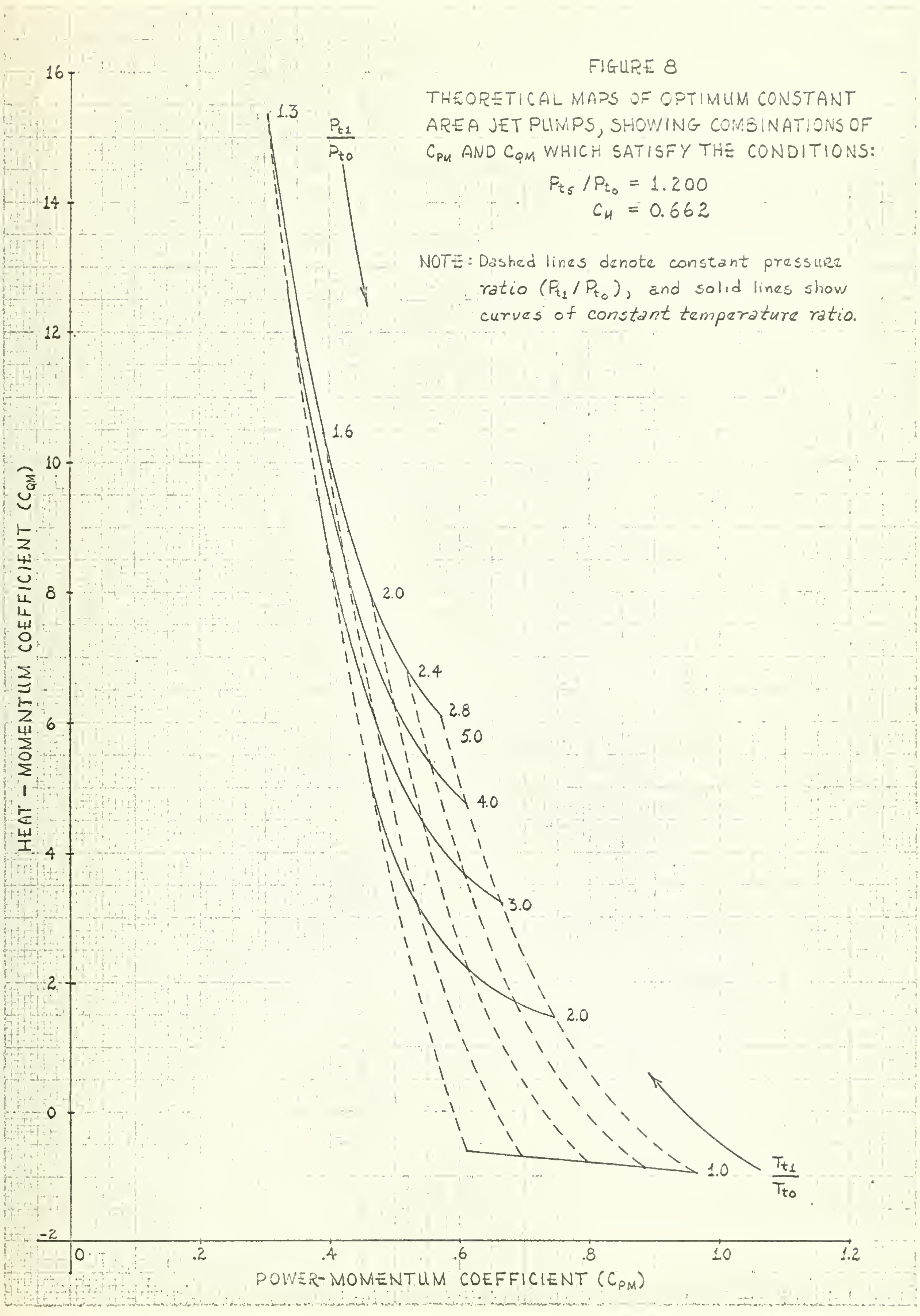


FIGURE 8

THEORETICAL MAPS OF OPTIMUM CONSTANT AREA JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:

$$P_{ts}/P_{t_o} = 1.200$$
$$C_M = 0.662$$

NOTE: Dashed lines denote constant pressure ratio (P_{t1}/P_{t_o}), and solid lines show curves of constant temperature ratio.

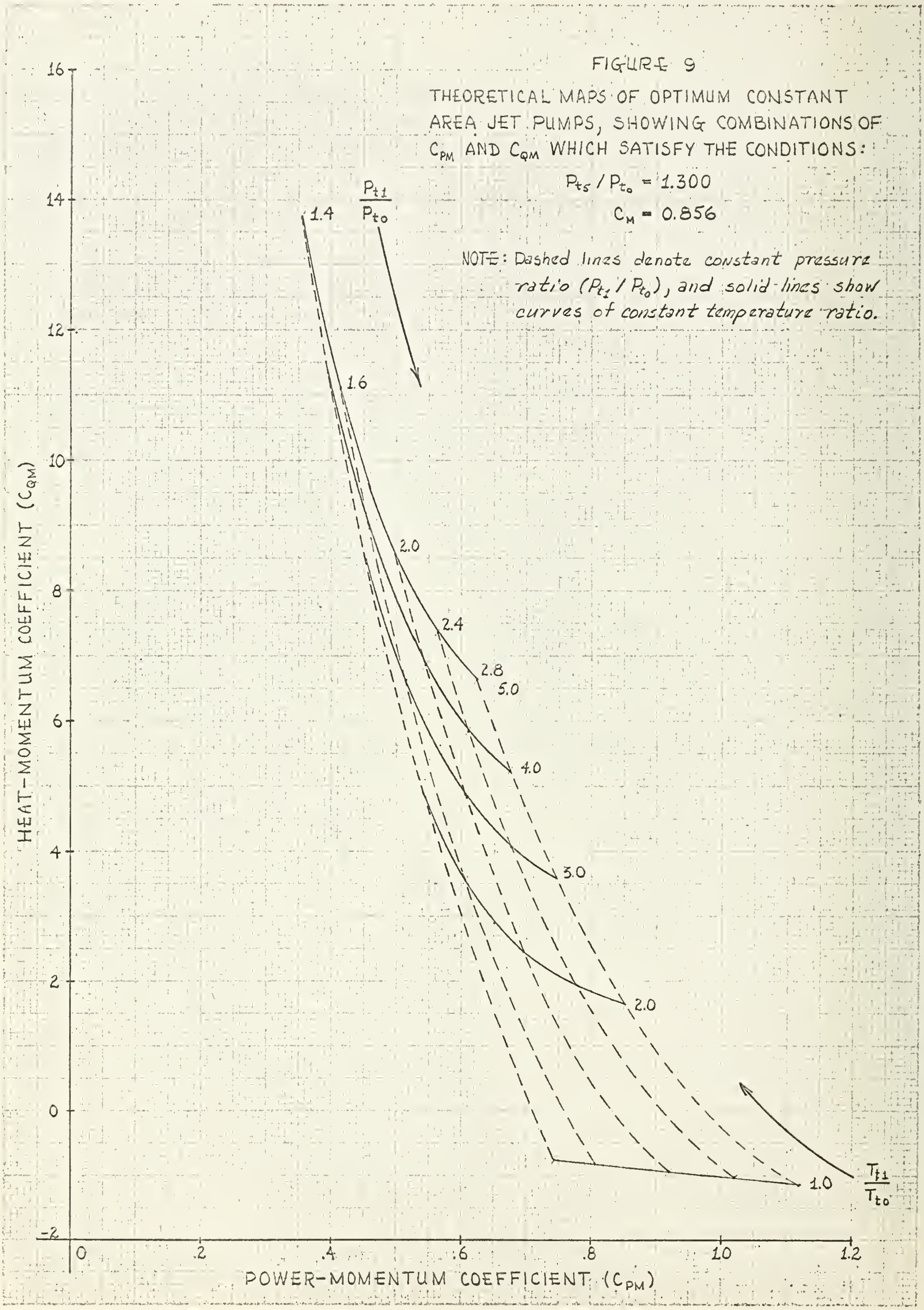


FIGURE 9

THEORETICAL MAPS OF OPTIMUM CONSTANT AREA JET PUMPS, SHOWING COMBINATIONS OF C_{PM} AND C_{QM} WHICH SATISFY THE CONDITIONS:

$P_{t5} / P_{t_0} = 1.300$

$C_M = 0.856$

FIGURE 10

PERFORMANCE MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS FOR P_{t_2}/P_{t_0} = 1.1 AND C_M = 0.442

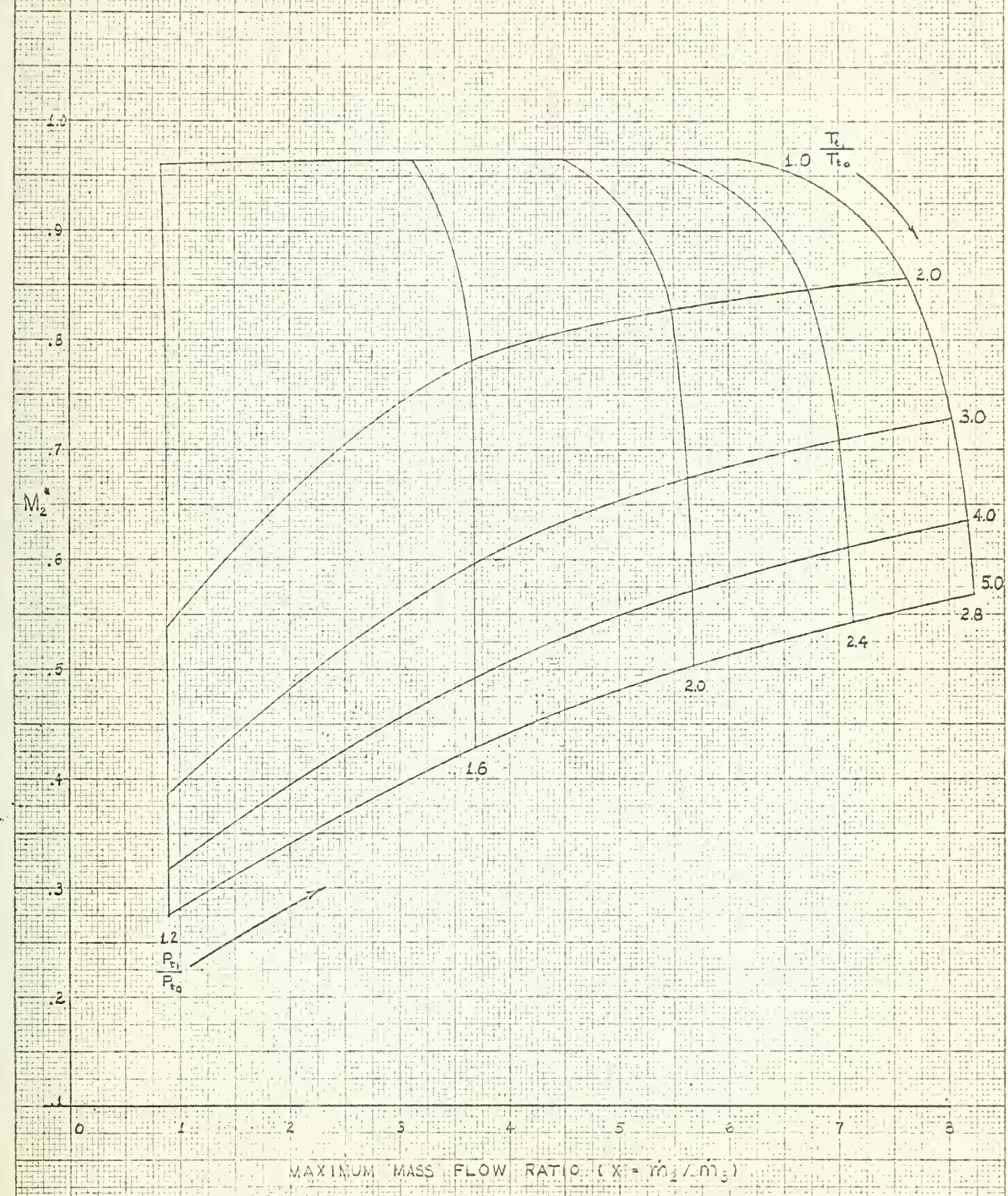


FIGURE 11

PERFORMANCE MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS FOR $P_{t5}/P_{t0} = 1.2$ AND $C_M = 0.662$

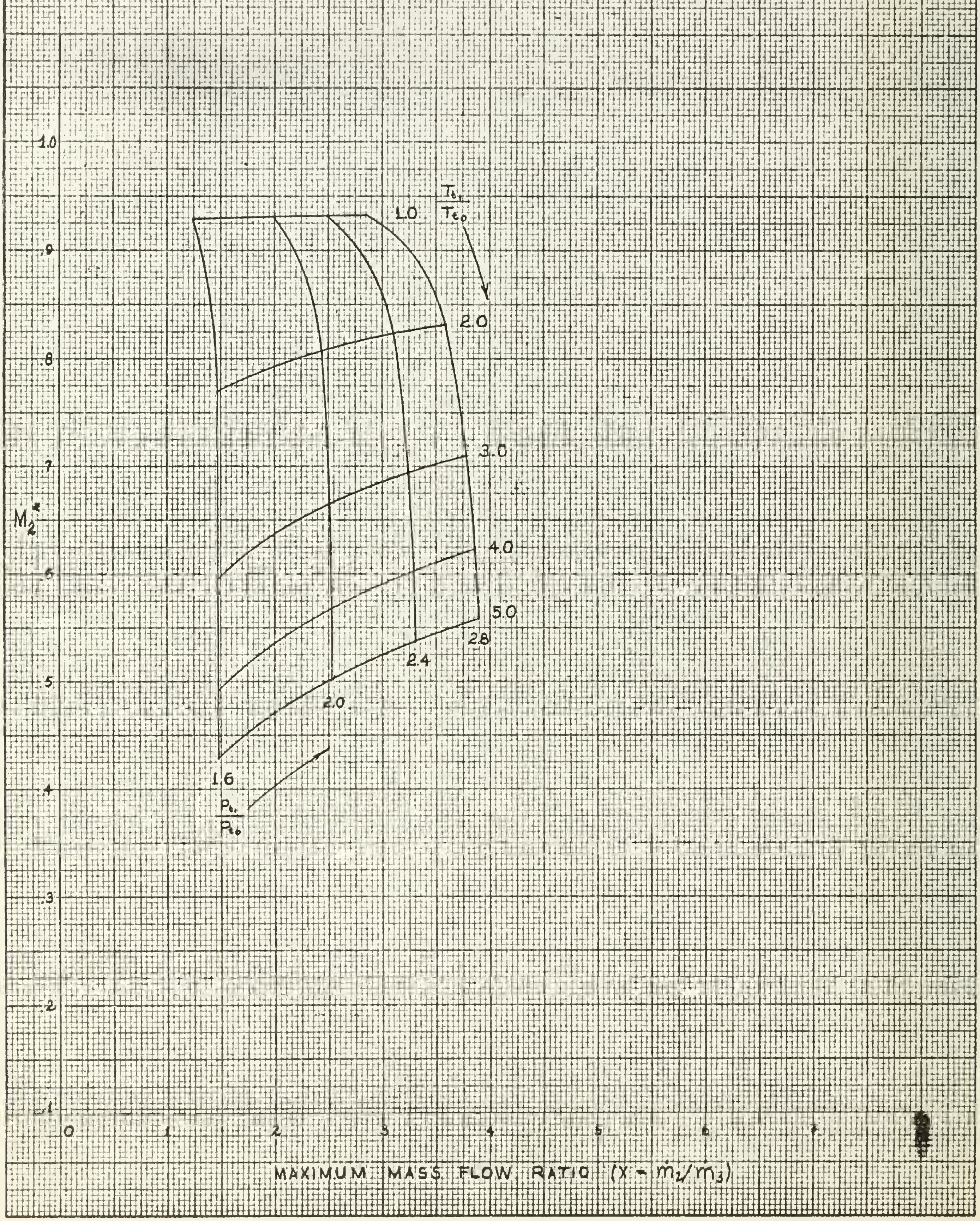


FIGURE 12

PERFORMANCE MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS FOR $P_{t5}/P_{t0} = 1.3$ AND $C_{M} = 0.856$

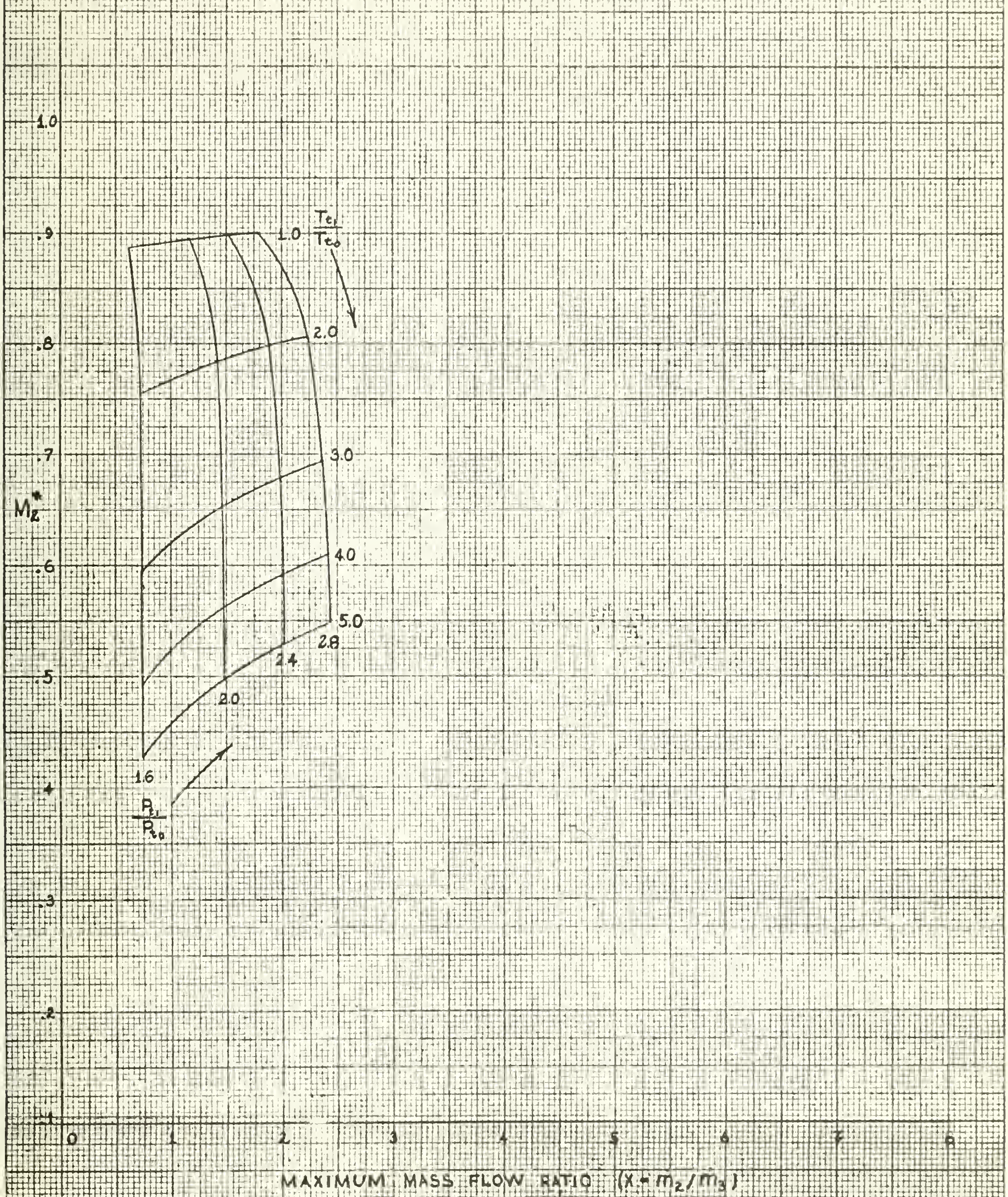


FIGURE 13
PERFORMANCE MAPS OF OPTIMUM CONSTANT AREA JET PUMPS FOR $P_{t_3}/P_{t_0} = 1.1$ AND $C_4 = 0.442$.

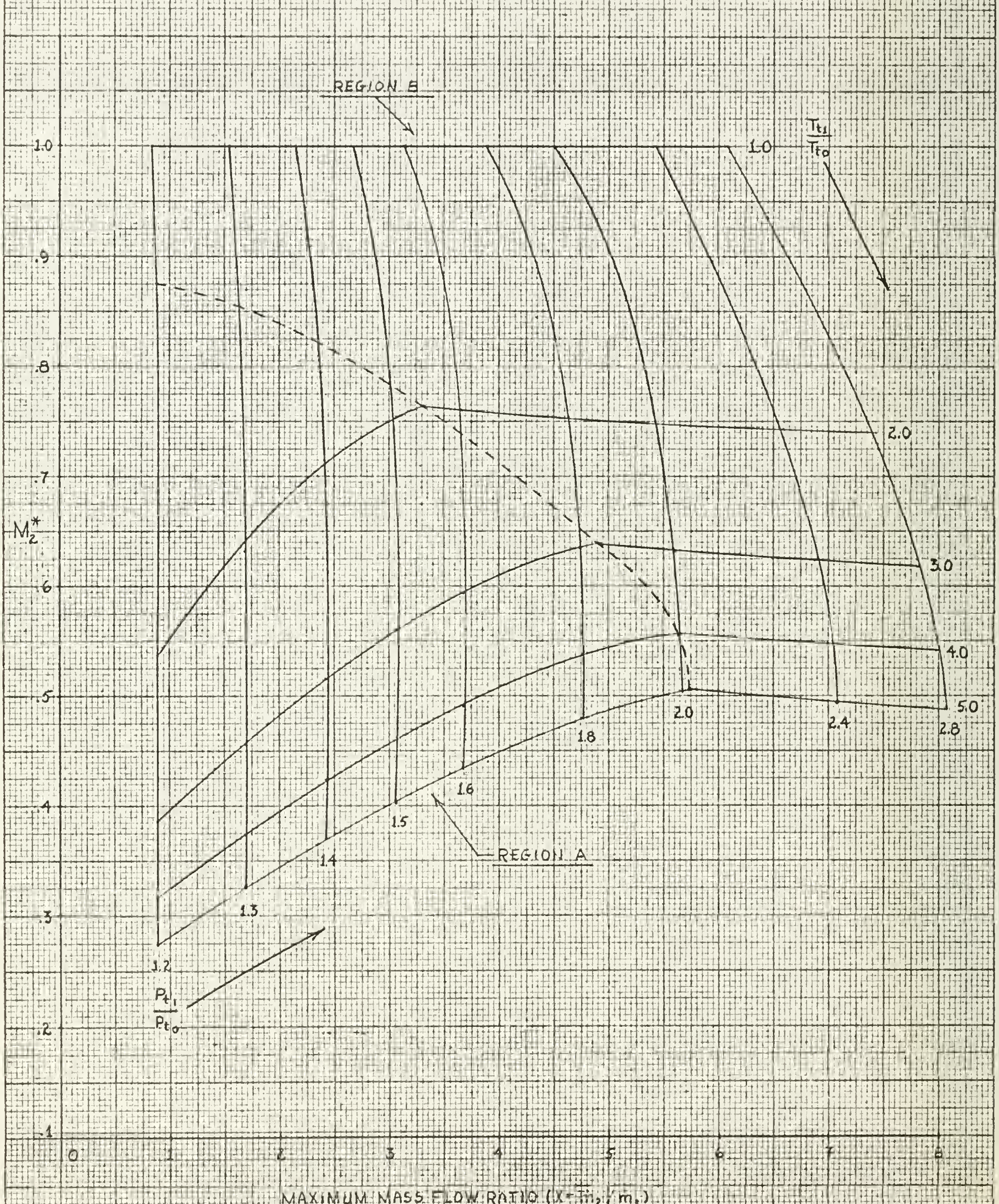


FIGURE 14

PERFORMANCE MAPS OF OPTIMUM CONSTANT AREA JET PUMPS FOR P_{t3}/P_{t0} = 1.2 AND C_{U} = 0.662.

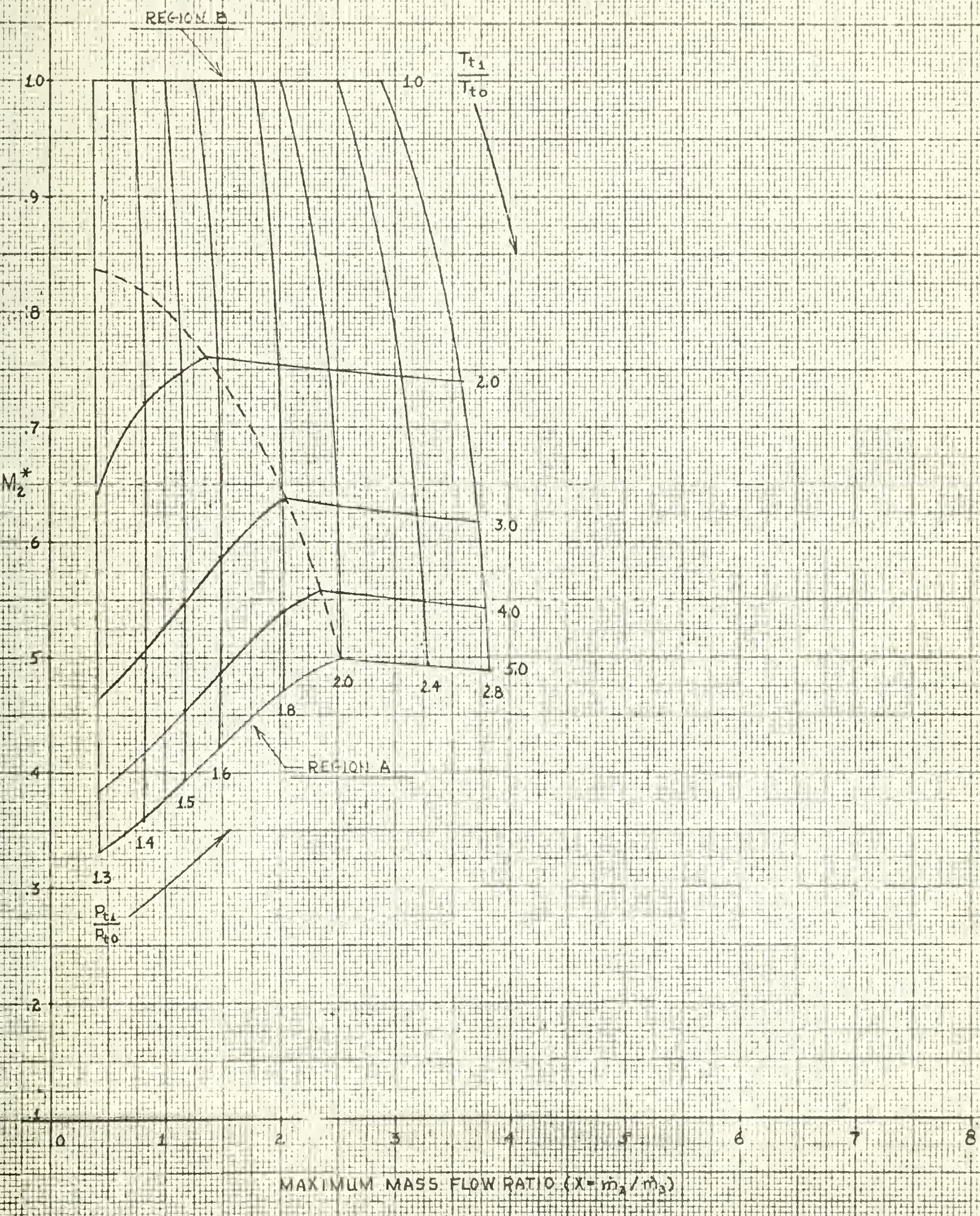
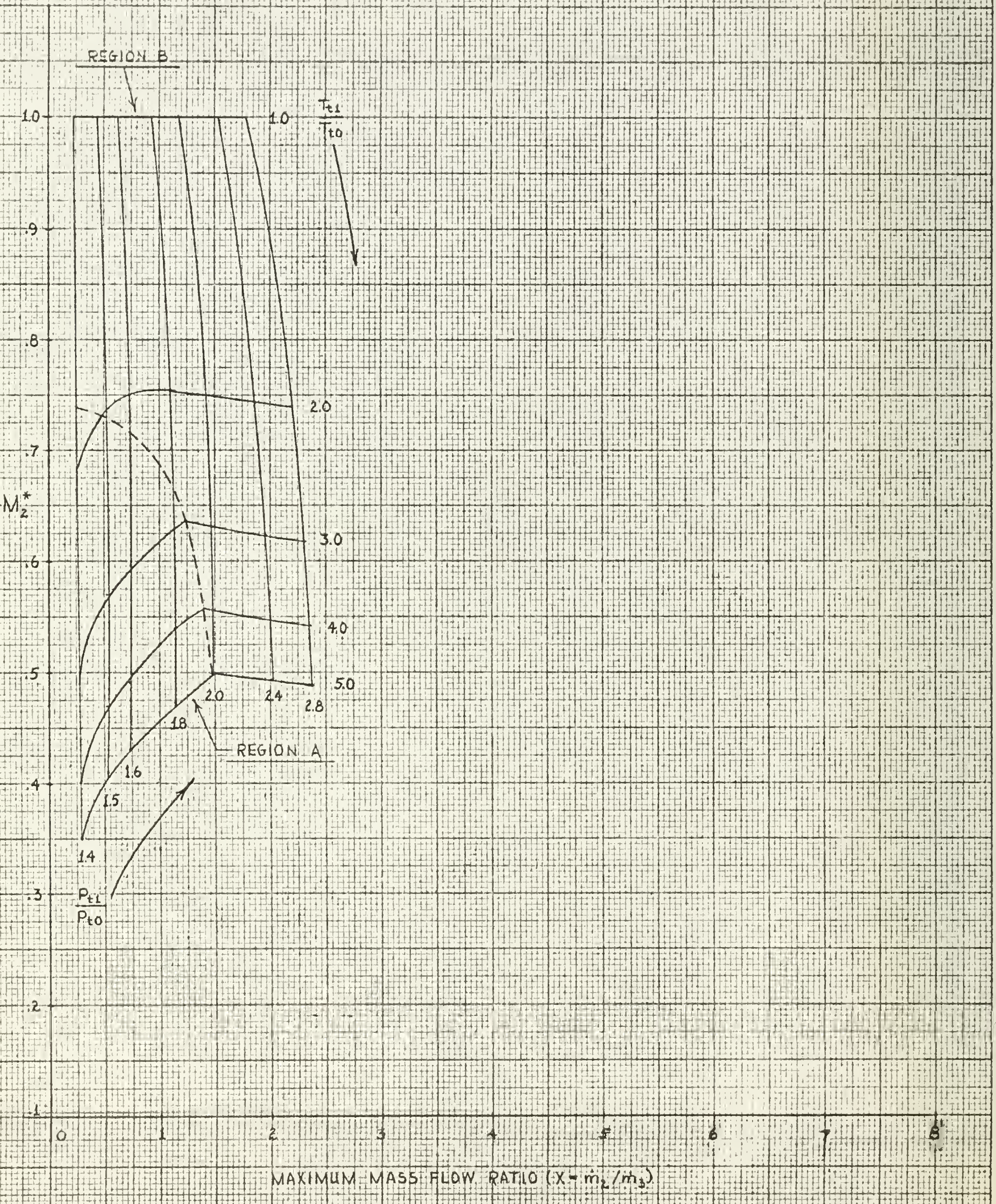


FIGURE 15
PERFORMANCE MAPS OF OPTIMUM CONSTANT AREA JET PUMPS FOR $P_{t5}/P_{t0} = 1.3$ AND $C_M = 0.856$



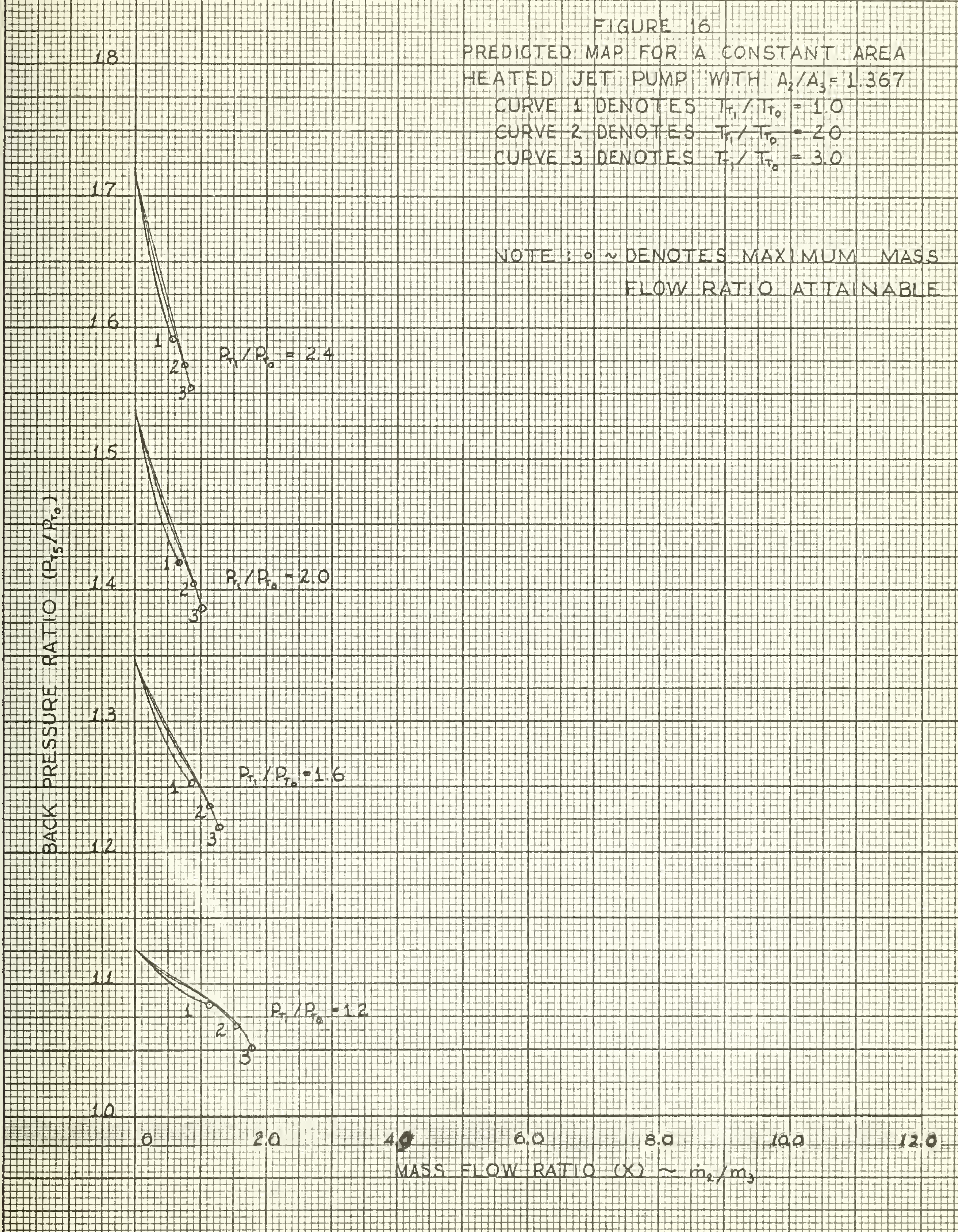


FIGURE 16
PREDICTED MAP FOR A CONSTANT AREA HEATED JET PUMP WITH $A_2/A_3 = 1.367$
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE : o ~ DENOTES MAXIMUM MASS FLOW RATIO ATTAINABLE
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
MASS FLOW RATIO (X) ~ $\dot{m}_2/m_3$
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
2.0
4.0
6.0
8.0
10.0
12.0

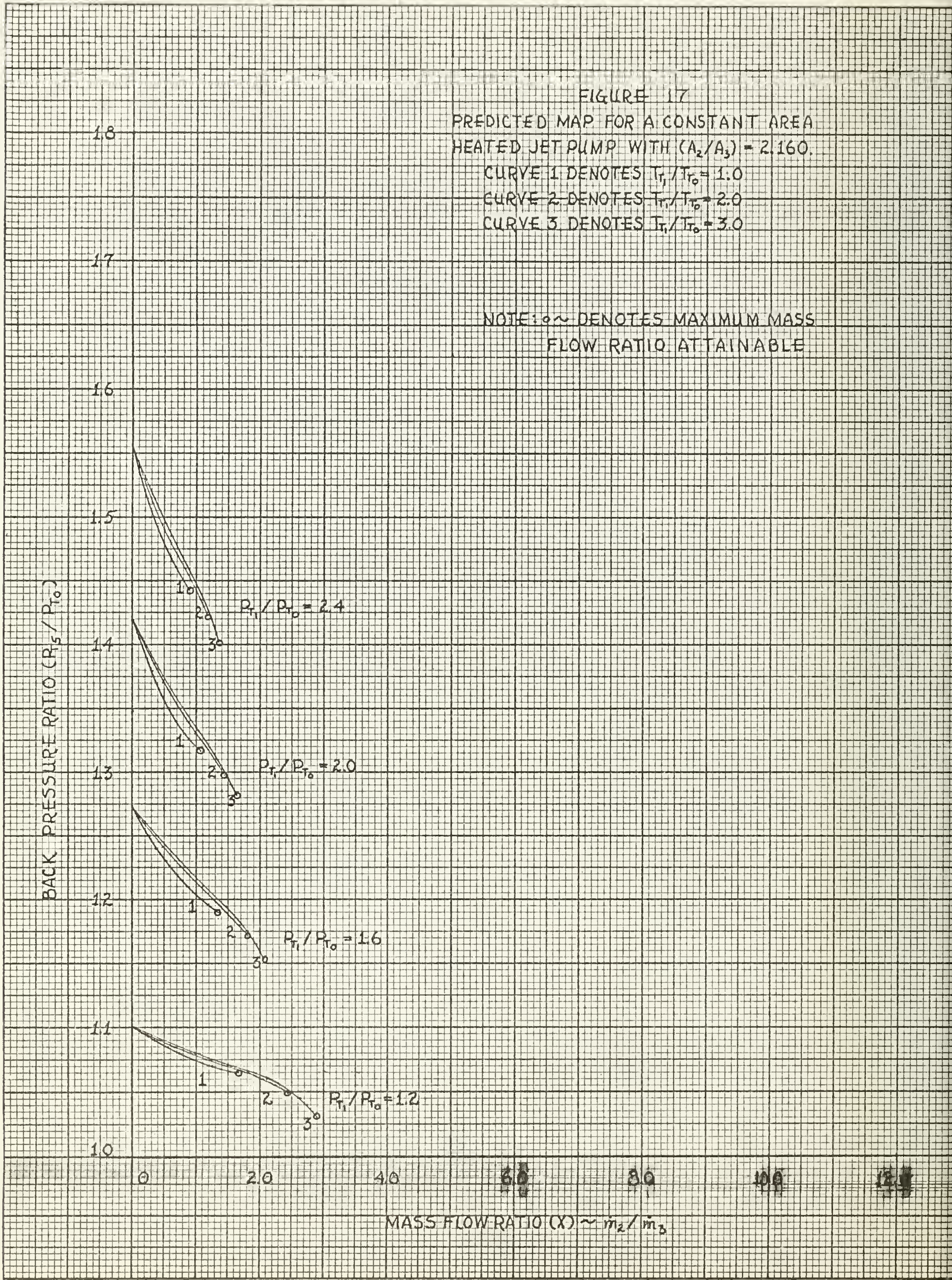


FIGURE 17
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH $(A_2/A_3) = 2.160$.
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE: o ~ DENOTES MAXIMUM MASS FLOW RATIO ATTAINABLE
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
1
2
3
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
2.0
4.0
6.0
8.0
10.0
12.0
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_3$

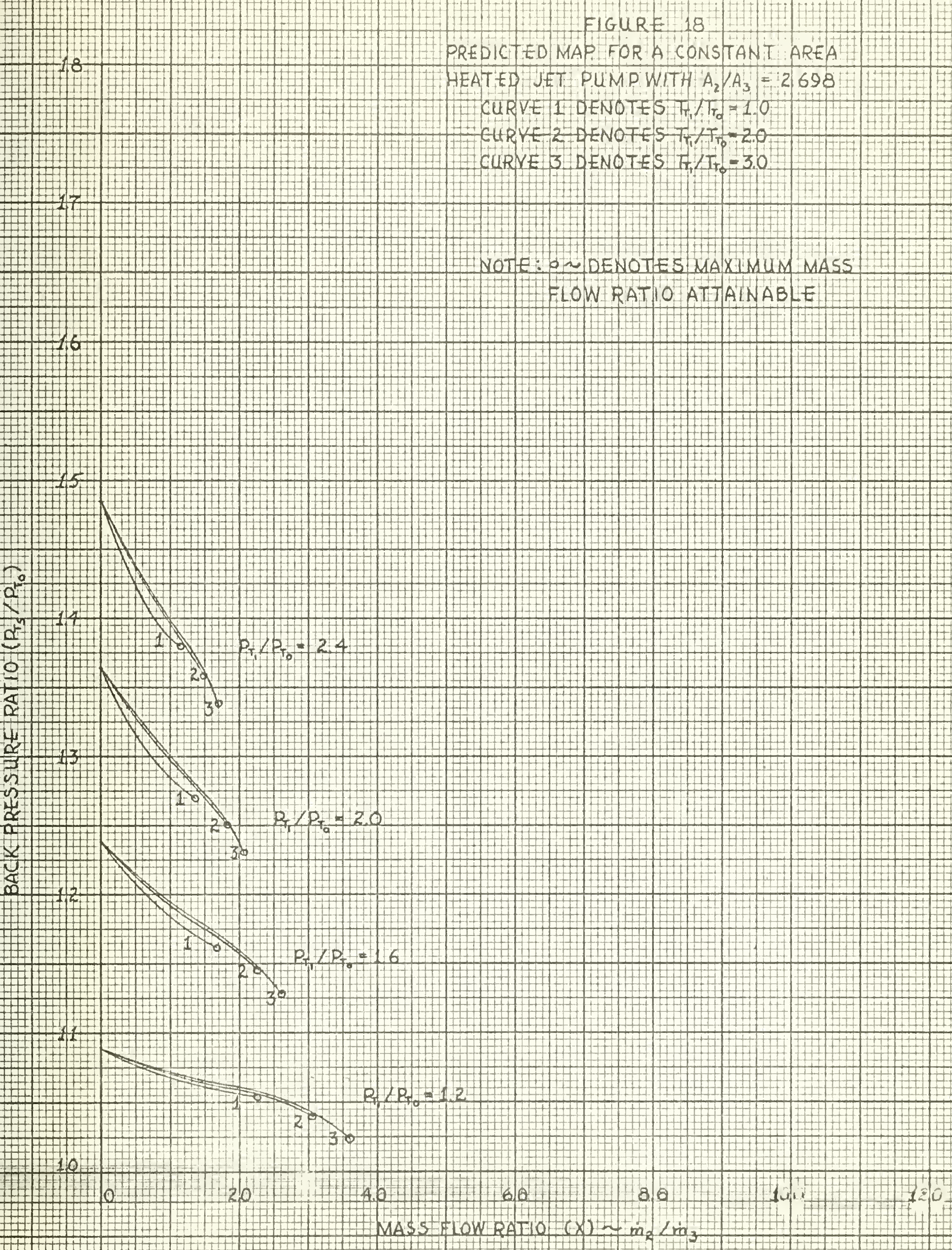


FIGURE 18
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH A_2/A_3 = 2.698
CURVE 1 DENOTES T_{T_1}/T_{T_0} = 1.0
CURVE 2 DENOTES T_{T_1}/T_{T_0} = 2.0
CURVE 3 DENOTES T_{T_1}/T_{T_0} = 3.0
NOTE: o ~ DENOTES MAXIMUM MASS FLOW RATIO ATTAINABLE
P_{T_1}/P_{T_0} = 2.4
P_{T_1}/P_{T_0} = 2.0
P_{T_1}/P_{T_0} = 1.6
P_{T_1}/P_{T_0} = 1.2
1
2
3
BACK PRESSURE RATIO (P_{T_3}/P_{T_0})
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
2.0
4.0
6.0
8.0
12.0
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_3$

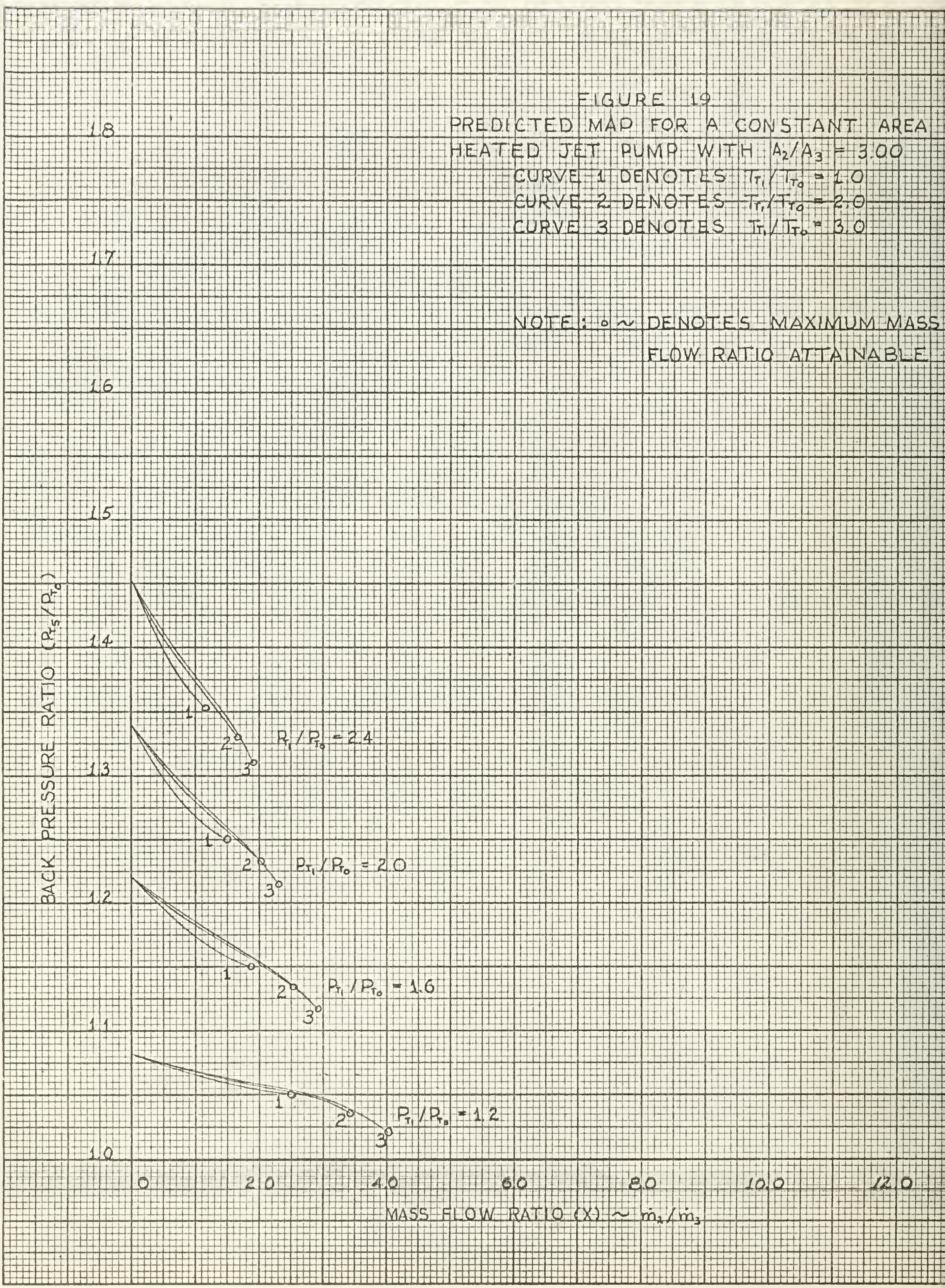


FIGURE 19
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH $A_2/A_3 = 3.00$
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE: o ~ DENOTES MAXIMUM MASS
FLOW RATIO ATTAINABLE
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
1
2
3
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
2.0
4.0
6.0
8.0
10.0
12.0
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_3$

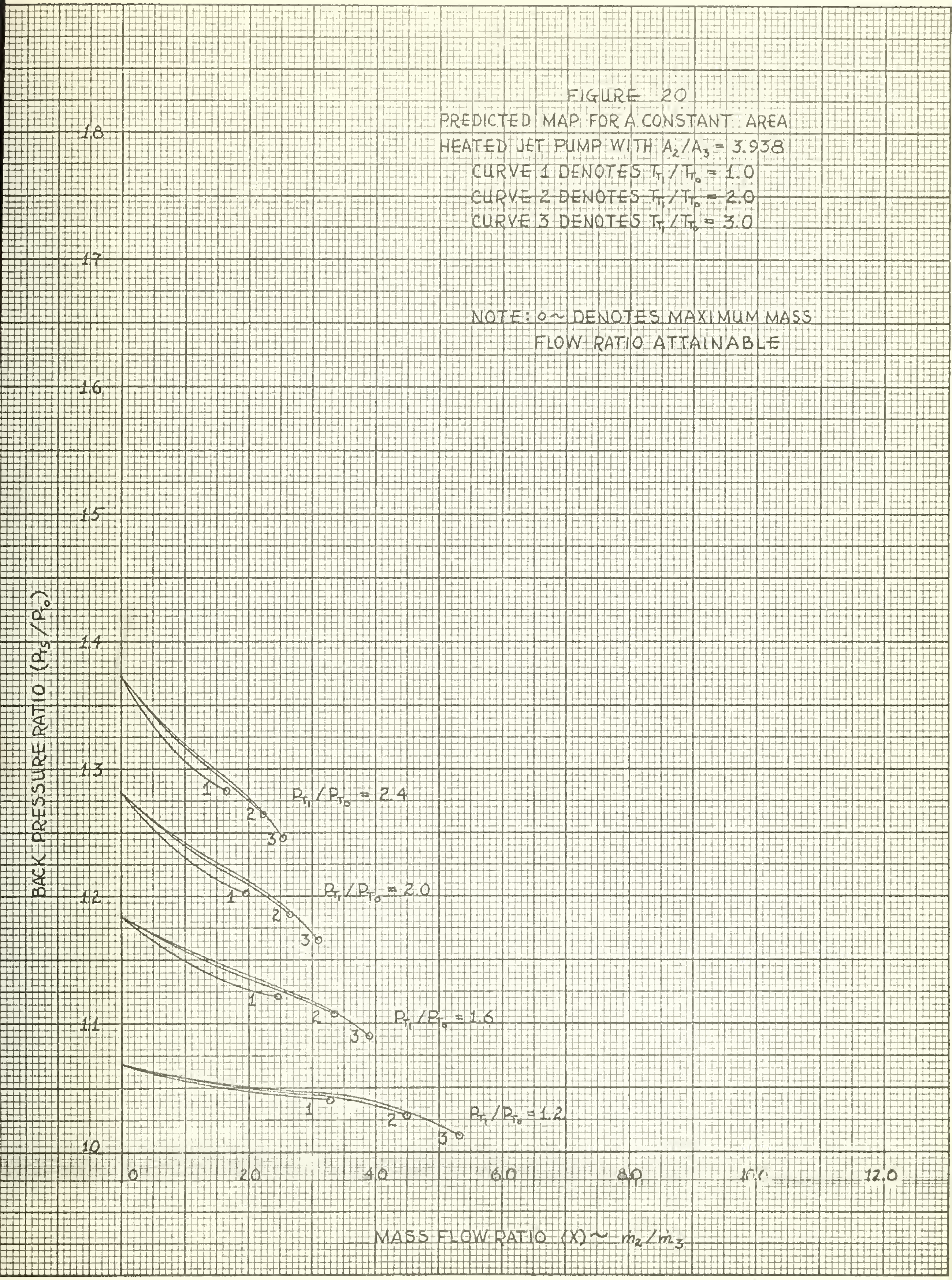


FIGURE 20

PREDICTED MAP FOR A CONSTANT AREA HEATED JET PUMP WITH $A_2/A_3 = 3.938$

CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$

CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$

CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$

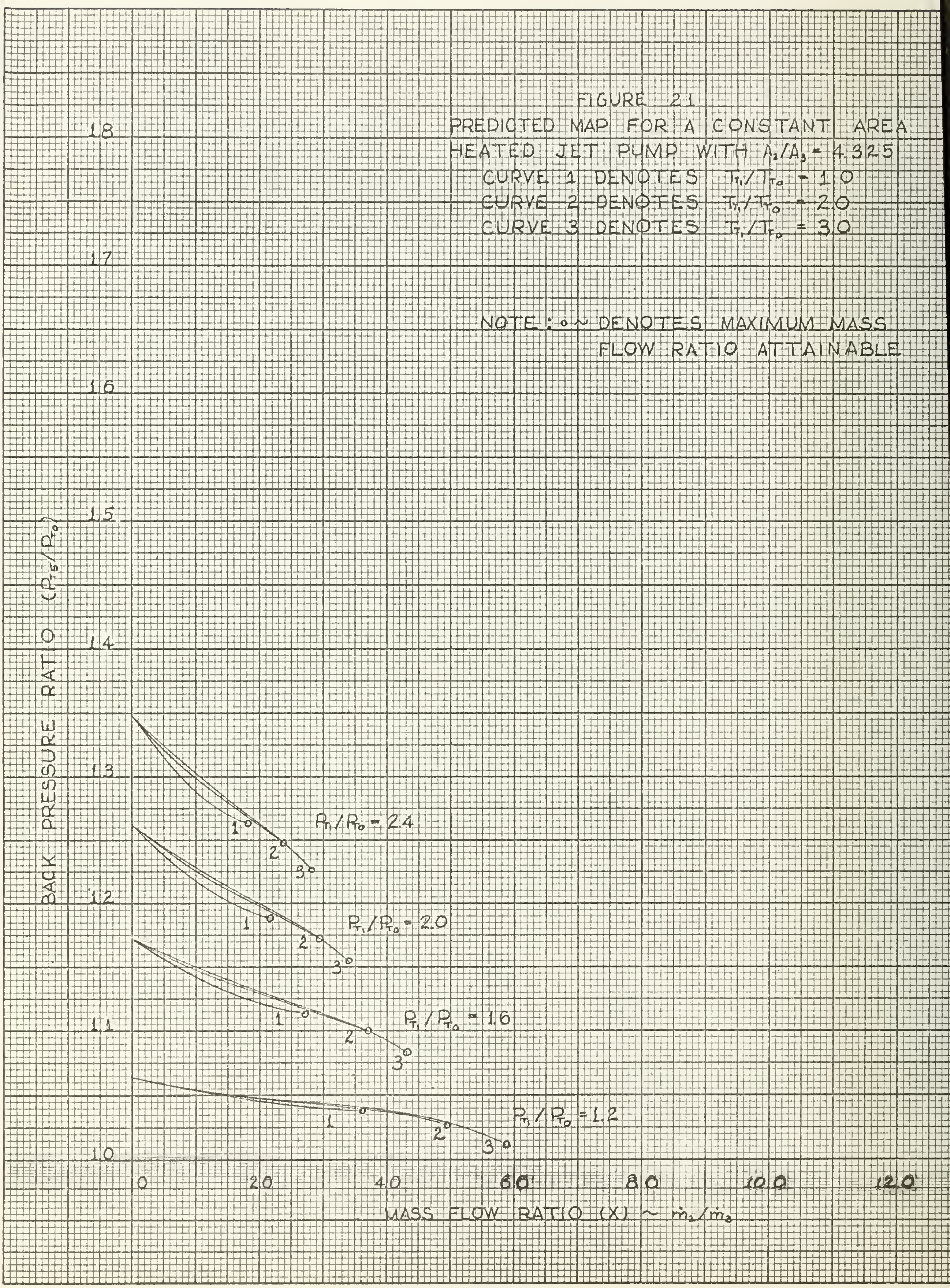


FIGURE 21
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH A_2/A_1 = 4.325
CURVE 1 DENOTES T_{T_1}/T_{T_0} = 1.0
CURVE 2 DENOTES T_{T_1}/T_{T_0} = 2.0
CURVE 3 DENOTES T_{T_1}/T_{T_0} = 3.0
NOTE : o ~ DENOTES MAXIMUM MASS
FLOW RATIO ATTAINABLE
P_{T_1}/P_{T_0} = 2.4
P_{T_1}/P_{T_0} = 2.0
P_{T_1}/P_{T_0} = 1.6
P_{T_1}/P_{T_0} = 1.2
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_0$
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
2.0
4.0
6.0
8.0
10.0
12.0

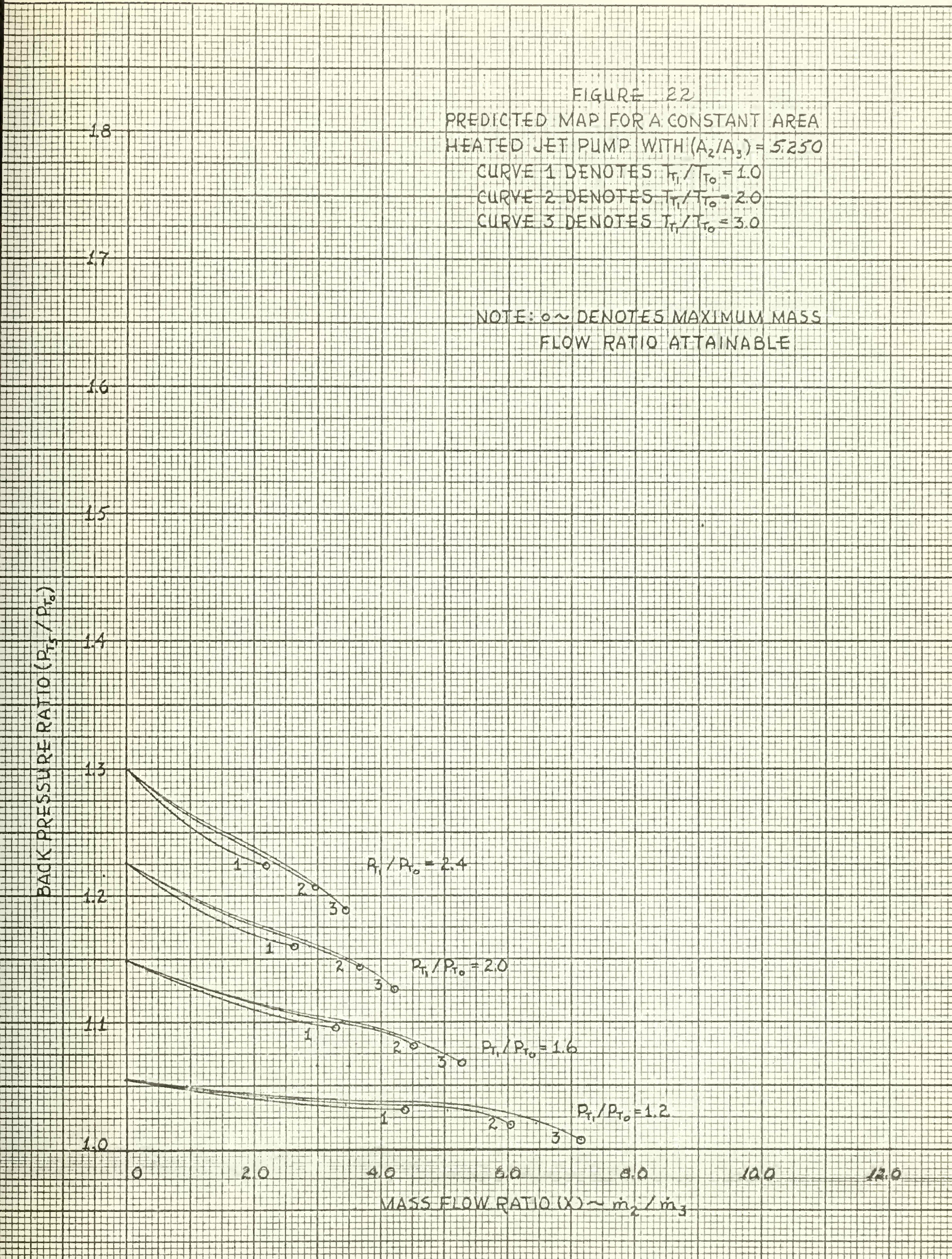


FIGURE 22
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH $(A_2/A_3) = 5.250$
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE: o ~ DENOTES MAXIMUM MASS FLOW RATIO ATTAINABLE
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
2.0
4.0
6.0
8.0
10.0
12.0
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_3$

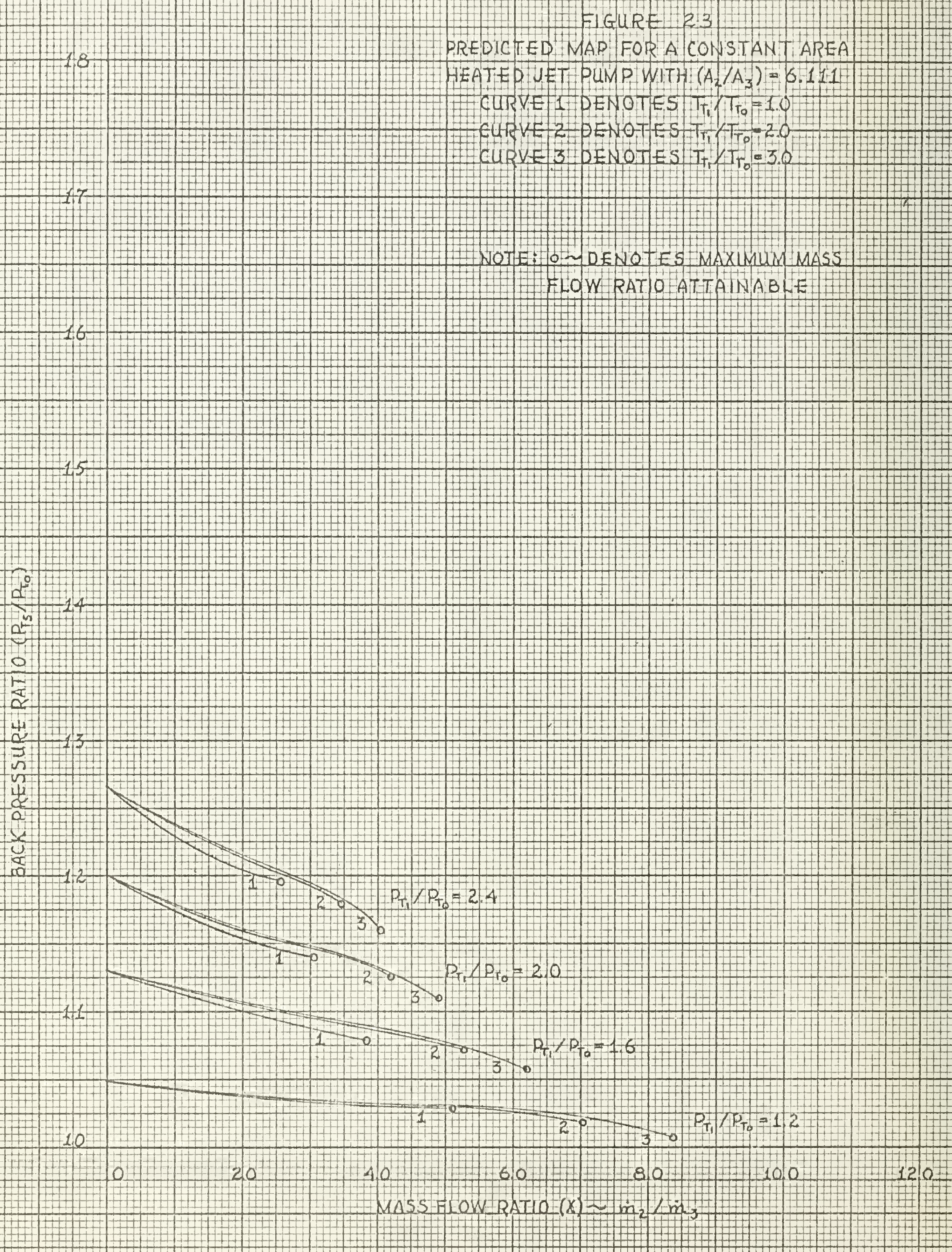


FIGURE 2.3
PREDICTED MAP FOR A CONSTANT AREA HEATED JET PUMP WITH $(A_2/A_3) = 6.111$
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE: o ~ DENOTES MAXIMUM MASS FLOW RATIO ATTAINABLE
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
BACK PRESSURE RATIO (P_{T_5}/P_{T_0})
MASS FLOW RATIO $(X) \sim \dot{m}_2/\dot{m}_3$
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
2.0
4.0
6.0
8.0
10.0
12.0

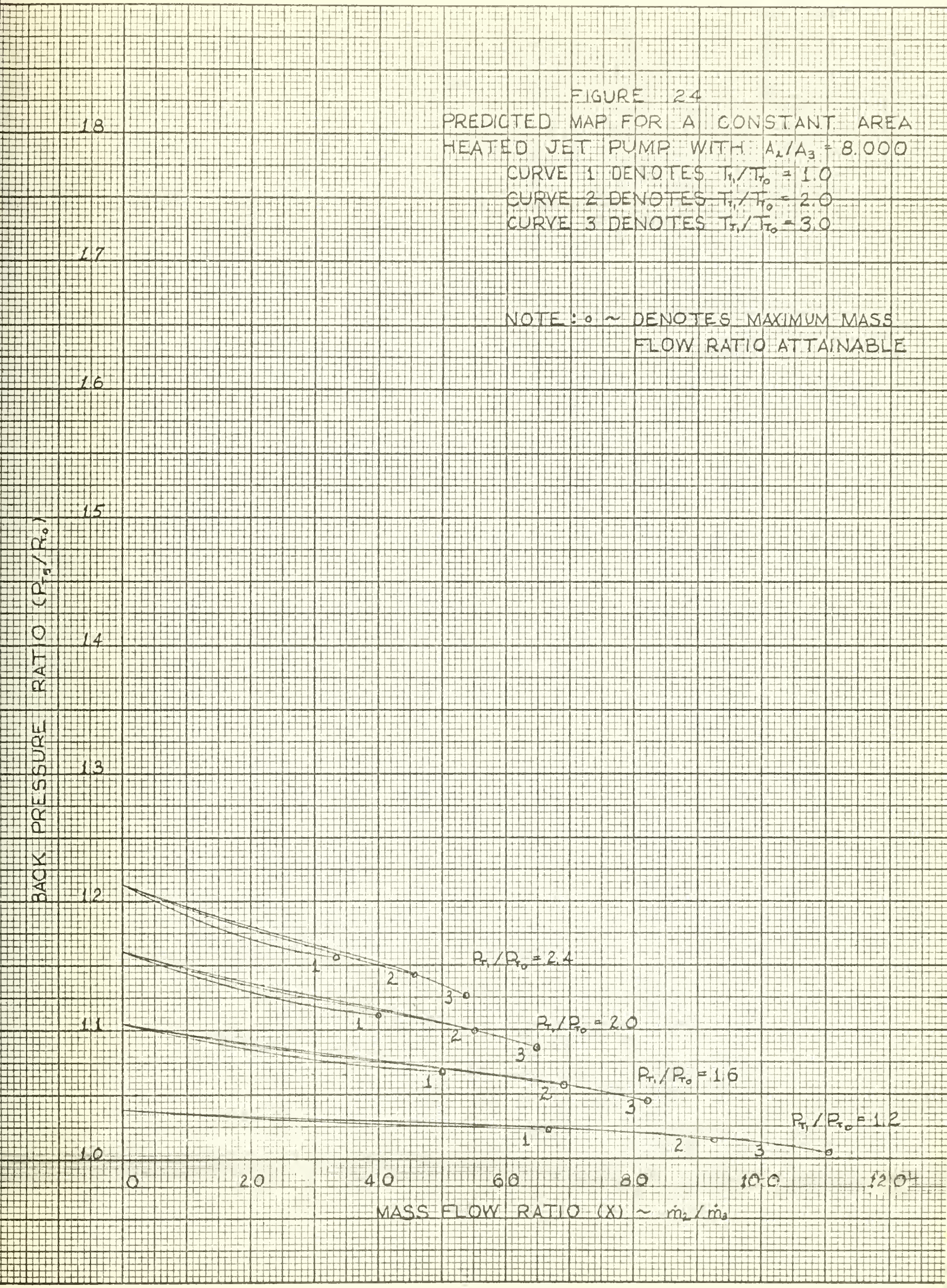
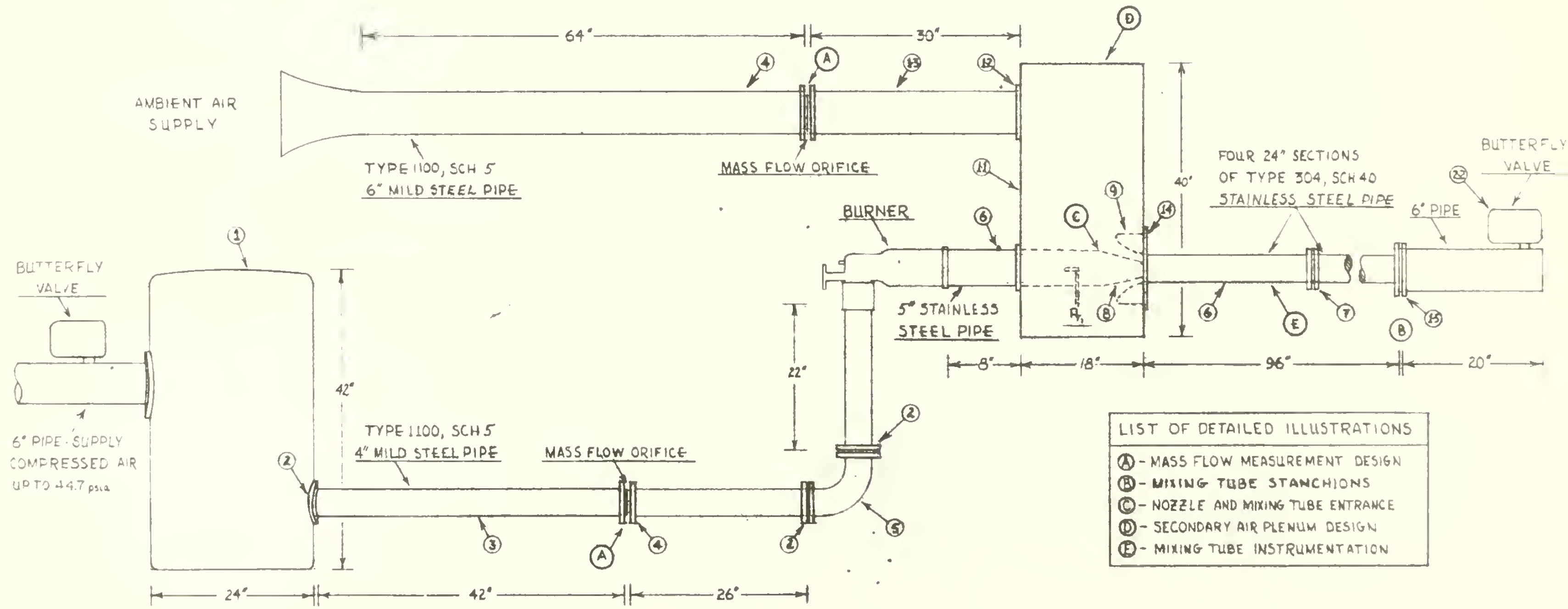


FIGURE 24
PREDICTED MAP FOR A CONSTANT AREA
HEATED JET PUMP WITH $A_2/A_3 = 8.000$
CURVE 1 DENOTES $T_{T_1}/T_{T_0} = 1.0$
CURVE 2 DENOTES $T_{T_1}/T_{T_0} = 2.0$
CURVE 3 DENOTES $T_{T_1}/T_{T_0} = 3.0$
NOTE: o ~ DENOTES MAXIMUM MASS
FLOW RATIO ATTAINABLE
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
BACK PRESSURE RATIO (P_{T_3}/P_{T_0})
$P_{T_1}/P_{T_0} = 2.4$
$P_{T_1}/P_{T_0} = 2.0$
$P_{T_1}/P_{T_0} = 1.6$
$P_{T_1}/P_{T_0} = 1.2$
1
2
3
0
2.0
4.0
6.0
8.0
10.0
12.0
MASS FLOW RATIO (X) ~ $\dot{m}_2/\dot{m}_3$

RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 25
JAN 1966	GENERAL DESIGN CONCEPT FOR A CONSTANT AREA HEATED JETPUMP	SCALE 16:1



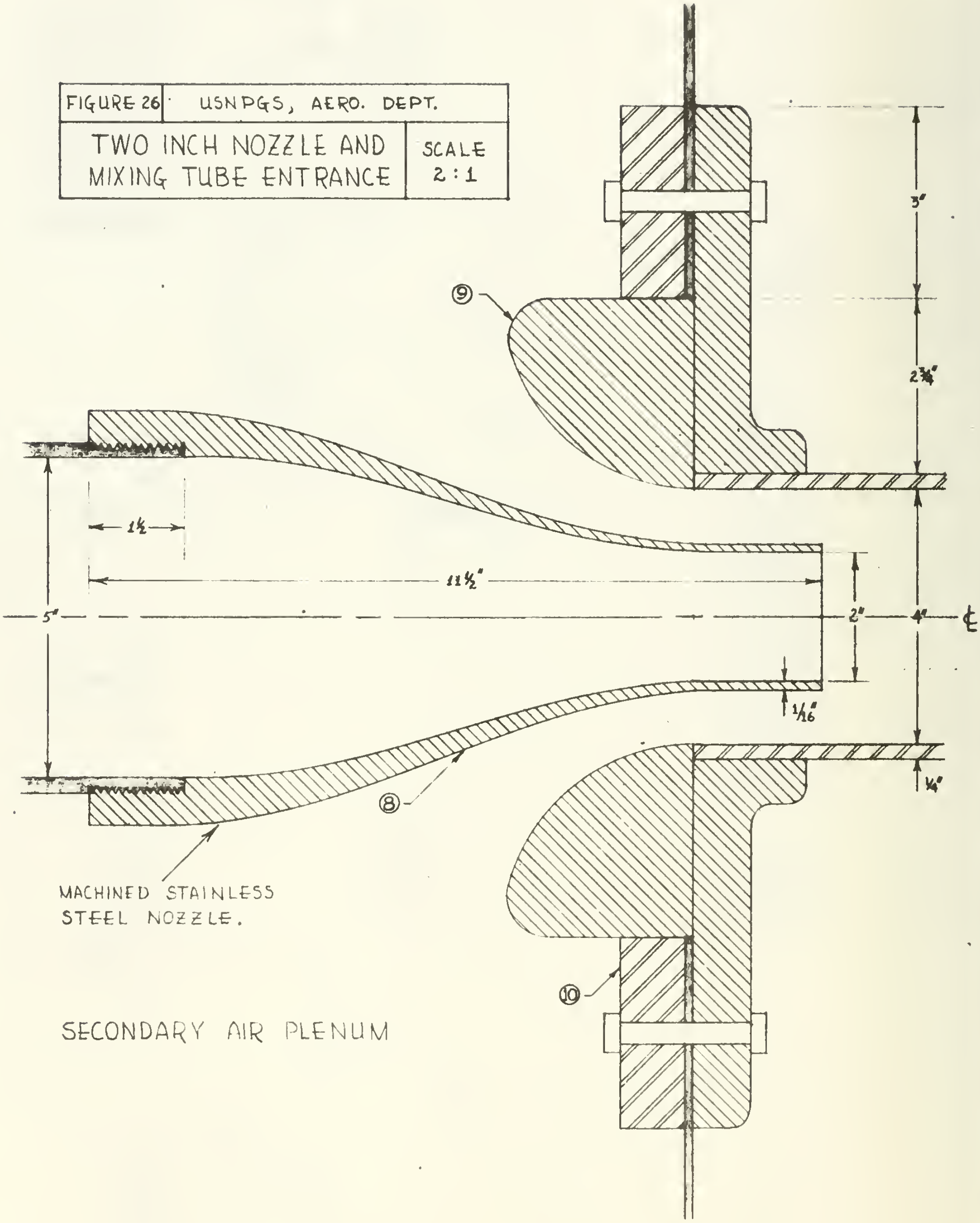


FIGURE 26
USNPGS, AERO. DEPT.
TWO INCH NOZZLE AND MIXING TUBE ENTRANCE
SCALE 2:1
3"
2¾"
4"
1½
11½"
5"
2"
1/16"
¼"
℄
⑨
⑧
⑩
MACHINED STAINLESS STEEL NOZZLE.
SECONDARY AIR PLENUM

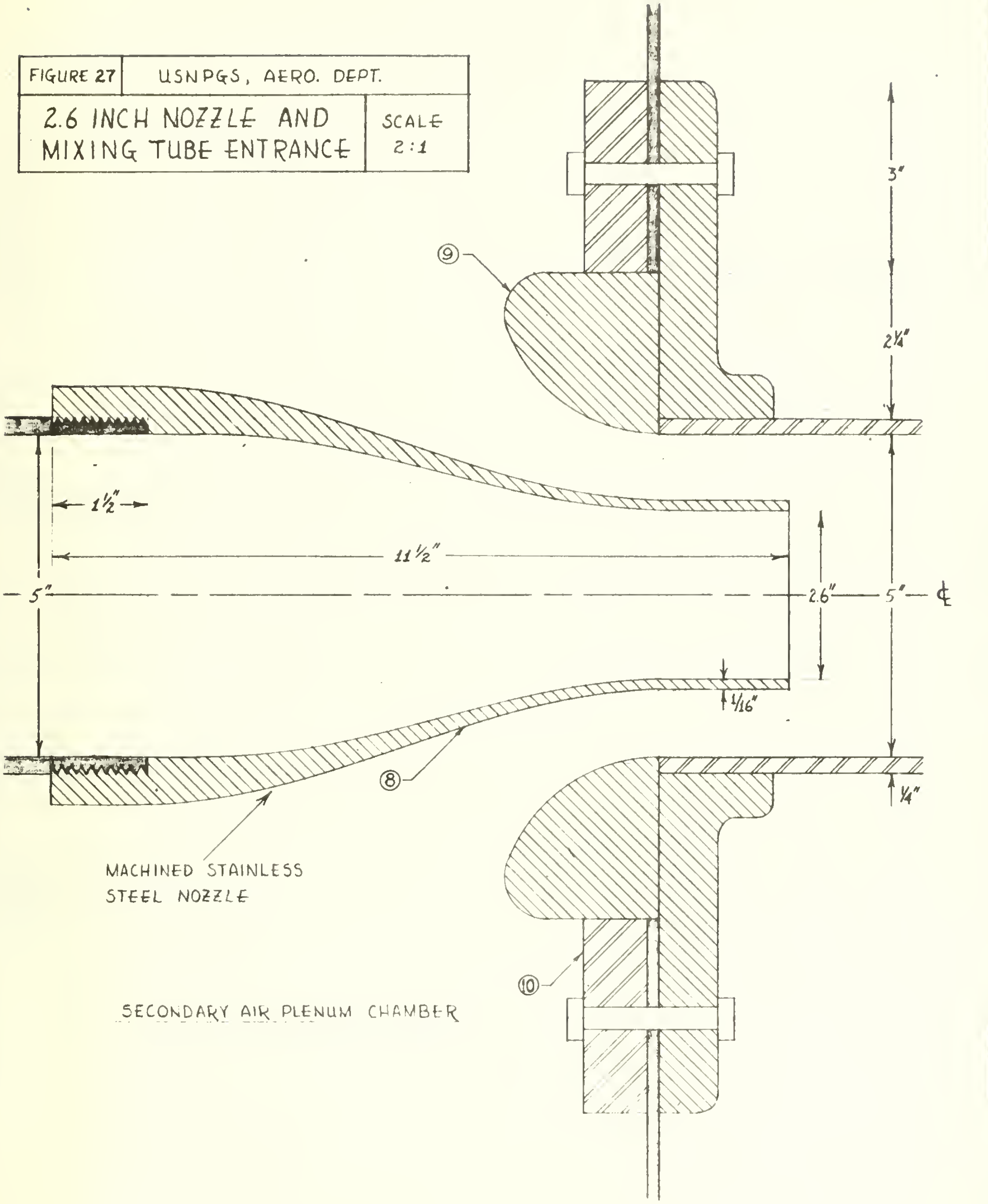


FIGURE 27 | USNPGS, AERO. DEPT.

2.6 INCH NOZZLE AND MIXING TUBE ENTRANCE | SCALE 2:1

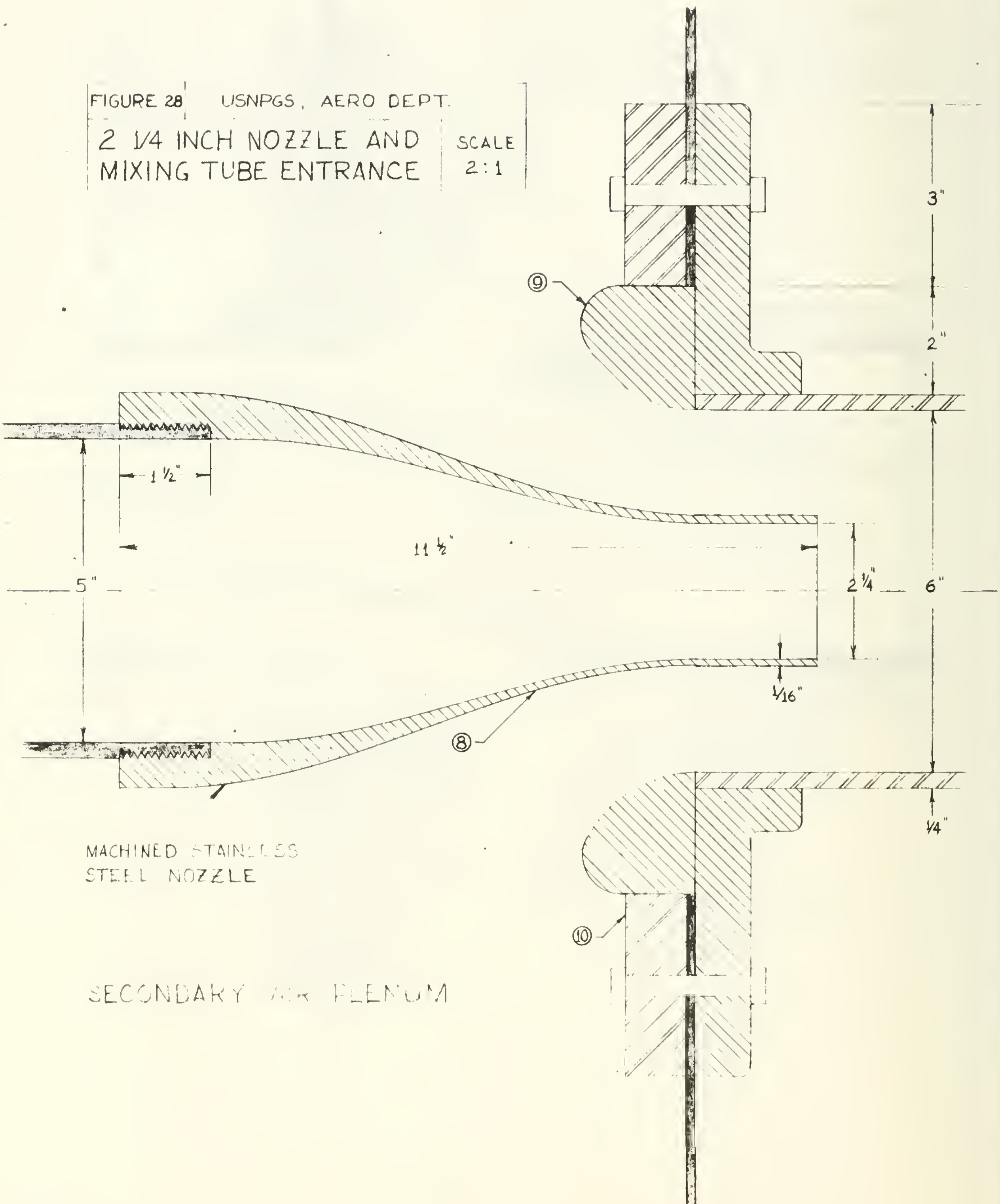
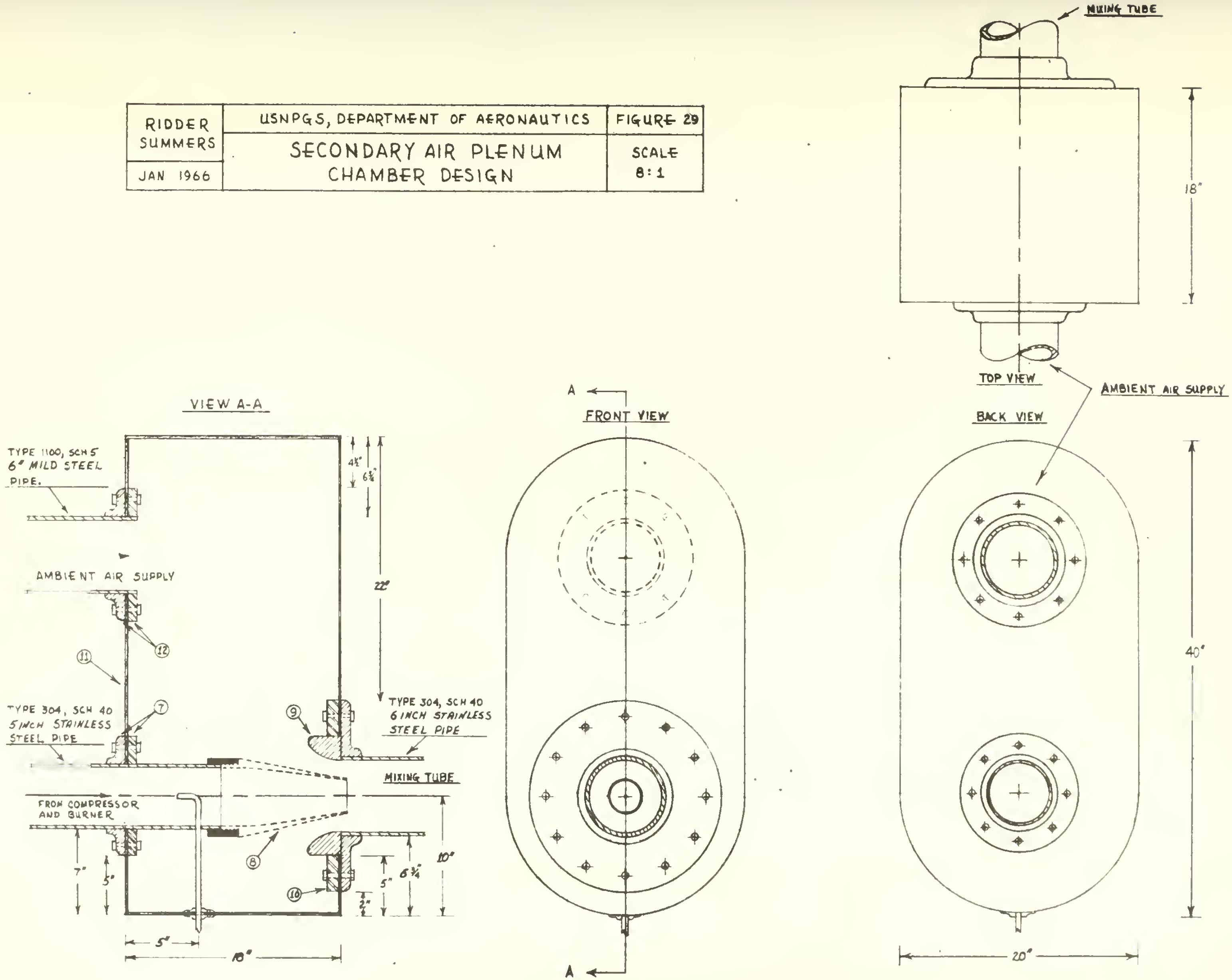


FIGURE 28 USNPGS, AERO DEPT.
2 1/4 INCH NOZZLE AND MIXING TUBE ENTRANCE
SCALE 2:1

RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 29
JAN 1966	SECONDARY AIR PLENUM CHAMBER DESIGN	SCALE 8:1



RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 30
JAN 1966	MIXING TUBE STANCHIONS AND SUPPORT DESIGN	SCALE 8:1

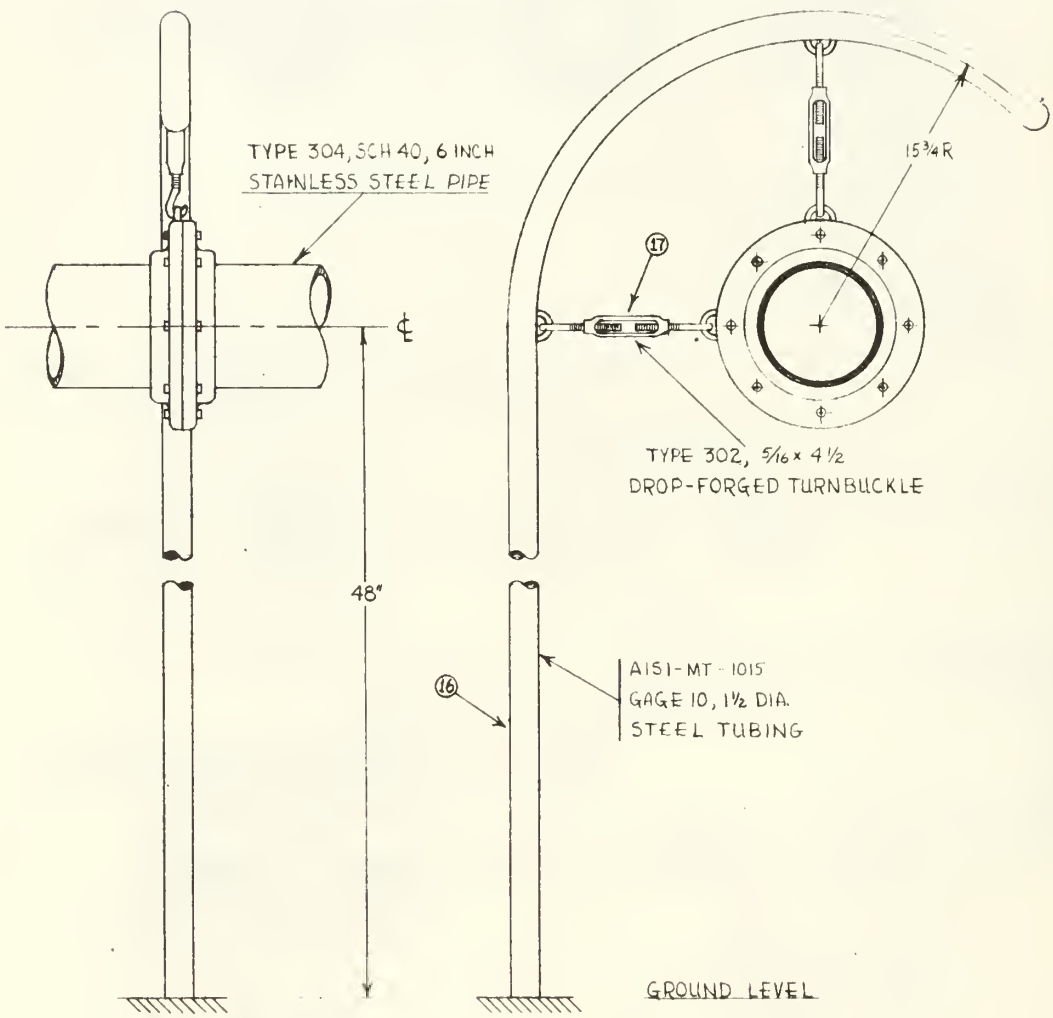
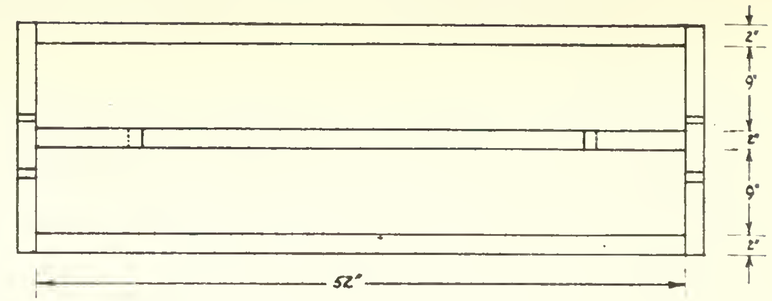
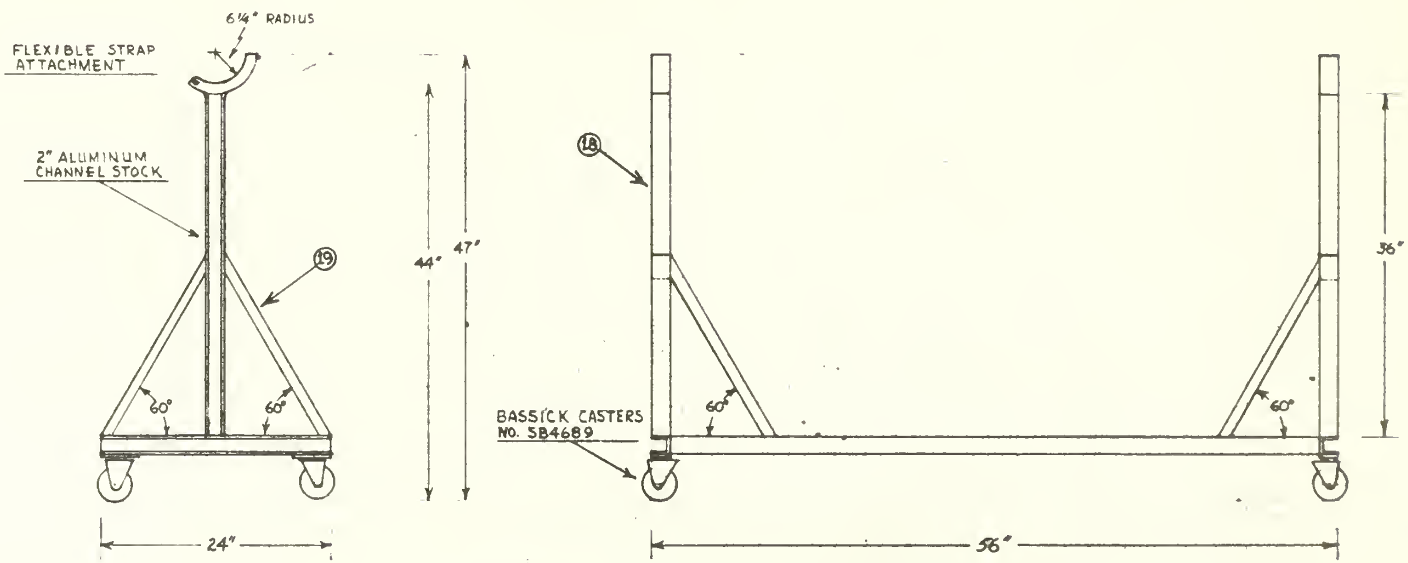
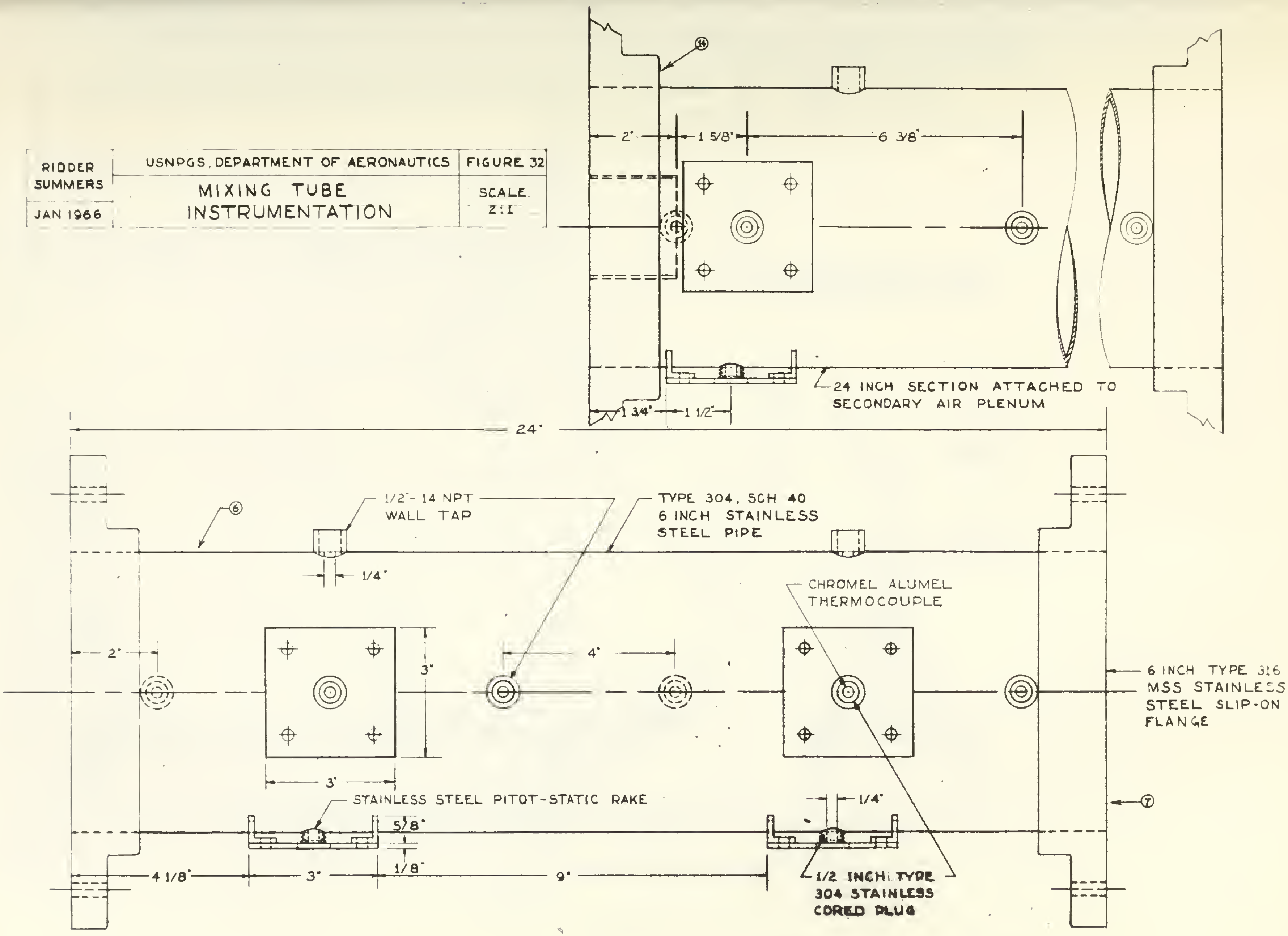


FIGURE 31	USNPGS, AERO DEPT.
MIXING TUBE HANDLING CART	SCALE 16 : 1



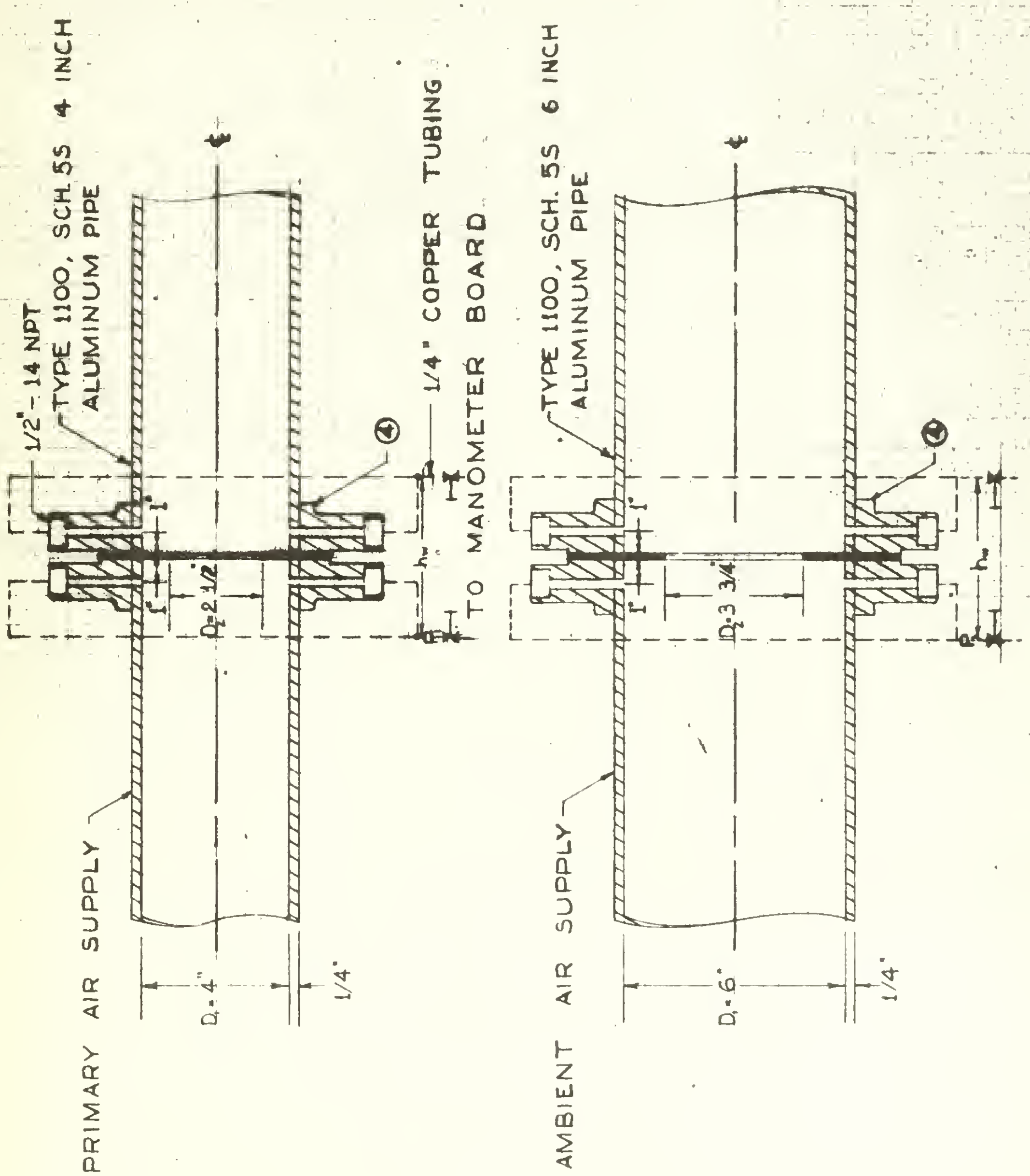
ALUMINUM CHANNEL AND BAR CONTRUCTION
MAXIMUM DESIGN LOAD 1000 lbs.
MAXIMUM LOAD ANTICIPATED 350 lbs.





RIDDER
SUMMERS
JAN 1966
USNPGS. DEPARTMENT OF AERONAUTICS
MIXING TUBE
INSTRUMENTATION
FIGURE 32
SCALE
2:1
2"
1 5/8"
6 3/8"
24 INCH SECTION ATTACHED TO SECONDARY AIR PLENUM
1 3/4"
1 1/2"
24"
1/2"- 14 NPT WALL TAP
TYPE 304, SCH 40 6 INCH STAINLESS STEEL PIPE
1/4"
CHROMEL ALUMEL THERMOCOUPLE
2"
3"
4"
3"
6 INCH TYPE 316 MSS STAINLESS STEEL SLIP-ON FLANGE
STAINLESS STEEL PITOT-STATIC RAKE
1/4"
5/8"
1/8"
4 1/8"
3"
9"
1/2 INCH TYPE 304 STAINLESS CORED PLUG

RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 38
JAN 1966	MASS FLOW MEASUREMENT DESIGN	SCALE 4:1



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APPENDIX A

Detailed Development

1. Preliminary relationships

The preliminary relationships are similar to those of Reference 1. All velocities are arbitrarily expressed in terms of the dimensionless velocity ratio, M^*. This ratio is obtained by dividing the flow velocity, V, by the reference velocity, a^*, which is the speed of sound at sonic velocity. The Mach number, M, is equal to M^* at sonic velocity and in all cases $aM = a^*M^*$. M^* is also proportional to a finite velocity at all absolute temperatures.

The following equations are used throughout the development:

$$a = \sqrt{\gamma g R T}$$

$$a^* = \sqrt{\frac{2\gamma}{\gamma+1} g R T_t} = \sqrt{\gamma g R T^*}$$

$$\frac{T}{T_t} = 1 - \frac{\gamma-1}{\gamma+1} M^{*2}$$

$$\frac{p}{P_t} = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_T} = \left(\frac{T}{T_t}\right)^{\frac{1}{\gamma-1}}$$

$$\dot{m}_2 = \rho_2 A_2 V_2$$

$$\dot{m}_3 = \rho_3 A_3 V_3$$

$$c_p = \frac{R}{J} \frac{\gamma}{\gamma - 1}$$

$$\gamma = \frac{c_p}{c_v}$$

$$p = \rho g R T$$

2. Performance coefficients

a. Momentum coefficient

The momentum coefficient is defined as the dimensionless momentum:

$$C_M = \frac{\rho_5 A_5 V_5^2}{\rho_o^* A_6 a_o^{*2}} \tag{A-1}$$

Substituting $a_5^{*2} M_5^{*2}$ for V_5^*

$$C_M = \frac{\rho_5 A_5 a_5^{*2} M_5^{*2}}{\rho_o^* A_6 a_o^{*2}} \tag{A-2}$$

Now $\rho_5 a_5^{*2} M_5^{*2}$ can be written

$$\rho_5 a_5^{*2} M_5^{*2} = \frac{\rho_5}{\rho_5^*} \rho_5^* a_5^{*2} M_5^{*2} \tag{A-3}$$

where

$$\frac{\rho_5}{\rho_5^*} = \left(\frac{T_5}{T_5^*}\right)^{\frac{1}{\gamma-1}}$$

which can be written

$$\frac{\rho_5}{\rho_5^*} = \left(\frac{T_5/T_{t5}}{T_5^*/T_{t5}}\right)^{\frac{1}{\gamma-1}} \tag{A-4}$$

Substituting for T_5/T_{t5} and T_5^*/T_{t5} into (A-4)

$$\frac{\rho_5}{\rho_5^*} = \left(\frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}}\right)^{\frac{1}{\gamma-1}}$$

$\rho_5^* a_5^{*2}$ can be written in terms of the equation of state and the speed of sound at sonic velocity.

$$\rho_5^* a_5^{*2} = \left(\frac{p_5^*}{g R T_5^*}\right)\left(\gamma g R T_5^*\right)$$

which can be written

$$\rho_5^* a_5^{*2} = \gamma \left(\frac{p_5^*}{P_{t5}}\right) P_{t5}$$

Since p_5^* corresponds to static pressure at $M_5 = M_5^* = 1$, then,

$$\rho_5^* a_5^{*2} = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} P_{t5}$$

Substituting for ρ_5/ρ_5^* and $\rho_5^* a_5^{*2}$ into (A-3)

$$\rho_5 a_5^{*2} M_5^{*2} = \left(\frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}}\right)^{\frac{1}{\gamma-1}} \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}} P_{t5} M_5^{*2}$$

Similar to $\rho_5^* a_5^{*2}$

$$\rho_o^* a_o^{*2} = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} P_{t_o}$$

Dividing $\rho_5 a_5^* M_5^*$ by $\rho_o^* a_o^{*2}$

$$\frac{\rho_5 a_5^* M_5^{*2}}{\rho_o^* a_o^{*2}} = \left(\frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}}\right)^{\frac{1}{\gamma-1}} \frac{P_{t5}}{P_{to}} M_5^{*2}$$

From continuity and remembering $M_6^* = 1$

$$\rho_5 A_5 V_5 = \rho_6 A_6 V_6$$

$$\frac{A_5}{A_6} = \frac{\rho_6 V_6}{\rho_5 V_5}$$

Substituting for V_5 and V_6

$$\frac{A_5}{A_6} = \frac{\rho_6 M_6^* a_6^*}{\rho_5 M_5^* a_5^*} \tag{A-5}$$

Since $T_{t_6} = T_{t_5}$, therefore $a_6^* = a_5^*$, hence

$$\frac{A_5}{A_6} = \frac{\rho_6}{\rho_5 \; M_5^*}$$

Substituting for $(\rho_6/\rho_{t_6})/(\rho_5/\rho_{t_6})$ in (A-5) and noting $\rho_{t_6} = \rho_{t_5}$

$$\frac{A_5}{A_6} = \left(\frac{\frac{2}{\gamma+1}}{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}} \right)^{\frac{1}{\gamma-1}} \frac{1}{M_5^*}$$

Substituting into Equation (A-2) for

$$\frac{\rho_5 \, a_5^* \, M_5^*}{\rho_0^* \, a_0^{*2}}$$

and A_5/A_6 , the result is

$$C_M = \frac{P_{t5}}{P_{to}} M_5^*$$

and substituting for M_5^*

$$C_M = \frac{P_{t5}}{P_{to}} \left\{ \frac{\gamma+1}{\gamma-1} \left[1 - \left(\frac{P_{to}}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

It is seen that C_M is a function only of the required discharge total pressure ratio. In particular, C_M is independent of the amount of heat supplied. By specifying the desired output pressure ratio, P_{t5}/P_{to} , the required C_M is determined.

b. Power-momentum coefficient

The ratio of the power coefficient to the momentum coefficient defines the power-momentum coefficient.

$$C_{PM} = \frac{C_P}{C_M} = \frac{\rho_3 A_3 V_3 \; a_0^* \; c_p \Delta T_c}{\rho_5 A_5 V_5^2 \; \frac{a_0^{*2}}{2gJ}}$$

$$C_{PM} = \frac{\dot{m}_3}{\dot{m}_2 + \dot{m}_3} \; \frac{\Delta H_c \, 2gJ}{V_5 \, a_0^*}$$

$$C_{PM} = \frac{\Delta H_c \, 2gJ}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) V_5 \, a_o^*} \tag{A-6}$$

$$\Delta H_c = c_p \left(T_{t1c} - T_{to}\right)$$

$$\Delta H_c = c_p T_{to} \left\{ \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

Substituting for ΔH_c and V_5 in (A-6), then C_{PM} becomes

$$C_{PM} = \frac{c_p T_{to} \left\{ \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} 2gJ}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) a_5^* \, M_5^* \, a_o^*}$$

also

$$\frac{2gJ c_p T_{to}}{a_5^* \, a_o^*} = \frac{2gJR\gamma T_{to}}{\left(\frac{2\gamma}{\gamma+1} gRT_{t5}\right)^{1/2} \left(\frac{2\gamma}{\gamma+1} gRT_{to}\right)^{1/2} J(\gamma-1)} \tag{A-7}$$

Therefore

$$C_{PM} = \frac{\left\{ \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \frac{\gamma+1}{\gamma-1}}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) \sqrt{\frac{T_{t5}}{T_{to}}} \, M_5^*} \tag{A-8}$$

rearranging and substituting in (A-8) for T_{T5}/T_{To} from the energy equation (A-31)

$$\frac{C_{PM}}{\left\{ \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} \quad \frac{\frac{\gamma+1}{\gamma-1} \left(M_5^* \right.}{\left\{ (x+1)\left(x + \frac{T_{t1}}{T_{to}}\right) \right\}} \tag{A-9}$$

c. **Heat-momentum coefficient**

The ratio of the heat coefficient to the momentum coefficient

defines the heat-momentum coefficient.

$$C_{QM} = \frac{C_Q}{C_M} = \frac{\rho_3 A_3 V_3 \; a_o^*}{\rho_5 A_5 V_5^2} \; \frac{c_p \, \Delta T_B}{\frac{a_o^{*2}}{2gJ}}$$

$$C_{QM} = \frac{\dot m_3}{\dot m_2 + \dot m_3} \; \frac{\Delta q \, 2gJ}{V_5 \, a_o^*}$$

$$C_{QM} = \frac{\Delta q}{(x+1)} \; \frac{2gJ}{a_5^* M_5^* a_o^*}$$

$$\Delta q = c_p \left(T_{t_1} - T_{t_{1c}}\right)$$

$$\Delta q = c_p T_{t_o} \left(\frac{T_{t_1}}{T_{t_o}} - \frac{T_{t_{1c}}}{T_{t_o}}\right)$$

Since there is an isentropic relationship between state 0 and $1c$, then Δq becomes

$$\Delta q = c_p T_{t_o} \left\{ \frac{T_{t_1}}{T_{t_o}} - \left(\frac{P_{t_1}}{P_{t_o}}\right)^{\frac{\gamma-1}{\gamma}} \right\}$$

Following a similar substitution for $\dfrac{2gJc_p T_{t_o}}{a_5^* \, a_o^*}$ as in (A-7) C_{QM} becomes:

$$C_{QM} = \frac{\left\{ \frac{T_{t1}}{T_{to}} - \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} \right\} \frac{\gamma+1}{\gamma-1}}{(x+1)\sqrt{\frac{T_{t5}}{T_{to}}} \; M_5^*} \tag{A-10}$$

Rearranging and substituting for T_{t5}/T_{t_o} from the energy equation (A-31)

$$\frac{C_{QM}}{\left\{ \frac{T_{t1}}{T_{to}} - \left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} \right\}} = \frac{\frac{\gamma+1}{\gamma-1}\left(M_5^*\right)^{-1}}{\left\{ (x+1)\left(x + \frac{T_{t1}}{T_{to}}\right) \right\}^{1/2}} \tag{A-11}$$

It is noted that the following equation can be written involving C_{QM} and C_{PM} from (A-9) and (A-11).

$$\frac{C_{PM}}{\left\{\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}} = \frac{C_{QM}}{\left\{\frac{T_{t1}}{T_{to}}-\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\right\}} = \frac{\frac{\gamma+1}{\gamma-1}\left(M_5^*\right)^{-1}}{\left\{(\chi+1)(\chi+\frac{T_{t1}}{T_{to}})\right\}^{1/2}} \tag{A-12}$$

Consider the case where P_{t_1}/P_{t_o} , T_{t_1}/T_{t_o} , and P_{t5}/P_{t_o} are fixed. This equation shows that under these conditions C_{PM} and C_{QM} will be a minimum when the mass flow ratio, $\dot{m}_1/\dot{m}_3 = \chi$, is a maximum. Also, it is noted that this equation is invariant for both the constant pressure and constant area cases, although the maximum value of χ may differ in the two cases.

d. Efficiency

Another performance parameter of some interest is the mechanical efficiency which is, of course, based on energy consideration, namely,

$$\eta_M = \frac{\text{Final available energy (Isentropic)}}{\text{Initial available energy (Isentropic)}}$$

$$\eta_M = \frac{\dot{m}_5\,(H_5 - h_5)}{\dot{m}_3\,\Delta H_c} = \frac{\rho_5 A_5 V_5}{\rho_3 A_3 V_3} \cdot \frac{V_5^2/2gJ}{c_p\,\Delta T_c}$$

$$\eta_M = \frac{1}{\frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5}} \; \frac{V_5}{\frac{a_o^*}{V_5}\;\frac{c_p\,\Delta T_c}{a_o^*/2gJ}\;a_o^*} = \frac{V_5}{C_{PM}\,a_o^*}$$

$$\eta_M = \frac{a_5^*\,V_5}{C_{PM}\,a_o^*\,a_5^*} = \frac{1}{C_{PM}}\sqrt{\frac{T_{t5}}{T_{to}}}\;M_5^*$$

Substituting for C_{PM} from (A-8)

$$\eta_M = \frac{(\chi+1)\sqrt{\frac{T_{ts}}{T_{to}}}\ M_s^*}{\left\{\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}\frac{\gamma+1}{\gamma-1}}\sqrt{\frac{T_{ts}}{T_{to}}}\ M_s^*$$

Substituting for T_{ts}/T_{to} from the energy equation (A-31).

$$\eta_M = \frac{\left(\frac{\gamma-1}{\gamma+1}\right) M_s^{*2}}{\left\{\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}}\left(\chi + \frac{T_{t1}}{T_{to}}\right) \tag{A-13}$$

Since, $p_s = P_{to}$, then

$$\frac{\gamma-1}{\gamma+1} M_s^{*2} = 1 - \left(\frac{P_{to}}{P_{ts}}\right)^{\frac{\gamma-1}{\gamma}}$$

Substituting for $\frac{\gamma-1}{\gamma+1} M_s^*$ into (A-13)

Therefore:

$$\eta_M = \frac{\left\{1 - \left(\frac{P_{to}}{P_{ts}}\right)^{\frac{\gamma-1}{\gamma}}\right\}}{\left\{\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}}\left(\chi + \frac{T_{t1}}{T_{to}}\right) \tag{A-14}$$

This equation shows that η_M , like the other performance parameters, will be optimum when χ is a maximum for specified conditions of pressure and temperature.

3. Development of constant pressure jet pump theory

An explicit solution to the equations of continuity, energy, and momentum is possible when the following parameters are considered fixed:

P_{ts}/P_{to}	the ratio of the demanded discharge total pressure to the ambient pressure.
P_{t1}/P_{to}	the ratio of the primary (jet) total pressure to the ambient pressure.
T_{t1}/T_{to}	the ratio of the primary (jet) total temperature to the ambient temperature.
M_2^*	the dimensionless secondary velocity.

The usual isentropic relationships are used to solve for conditions in the primary (jet) system and in the secondary jet system up to the mixing tube entrance.

The critical value of M_2^* for choking of the primary nozzle can be determined from the isentropic relationships

$$\frac{p_3}{P_{t_1}} = \frac{p_3}{P_{t_3}} = \left[1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}\right]^{\frac{\gamma}{\gamma-1}} \tag{A-15}$$

$$\frac{p_2}{P_{t_0}} = \frac{p_2}{P_{t_2}} = \left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right]^{\frac{\gamma}{\gamma-1}} \tag{A-16}$$

For sub-critical flow, $M_3^* < 1.0$ and $M_2^* < M_{2cr}^*$, the primary flow is unchoked and $p_2 = p_3$. M^*_{2cr} is the critical secondary velocity ratio at which M_3^* becomes sonic for supply pressure ratios less 1.89. Multiplying p_2/P_{t_0} by P_{t_1}/p_3 from (A-15) and (A-16) results in

$$\frac{P_{t_1}}{P_{t_0}} = \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}\right\}^{\frac{\gamma}{\gamma-1}} \tag{A-17}$$

Solving for M_3^* from (A-17)

$$M_3^* = \left\{\frac{\gamma+1}{\gamma-1}\left[1 - \frac{\left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right]}{\left(\frac{P_{t_1}}{P_{t_0}}\right)^{\frac{\gamma-1}{\gamma}}}\right]\right\}^{\frac{1}{2}} \tag{A-18}$$

Also

$$\frac{p_3}{P_{t_0}} = \frac{p_2}{P_{t_0}} = \frac{p_2}{P_{t_2}} = \left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right]^{\frac{\gamma}{\gamma-1}}$$

At critical conditions, $M_3^* = 1$, $M_2^* = M_{2cr}^*$, and $p_2 = p_3$. It is possible to determine the critical M_2^* as a function of P_{t_1}/P_{t_0} from (A-17)

$$\frac{2}{\gamma+1}\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma-1}{\gamma+1} M_{2cr}^{*2}$$

$$M_{2cr}^{*} = \left\{\frac{\gamma+1}{\gamma-1}\left[1 - \frac{2}{\gamma+1}\left(\frac{P_{t1}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}} \qquad \text{(A-19)}$$

For super-critical flow, $M_3^* = 1$, $M_2^* > M_{2cr}^*$, and $p_2 \neq p_3$. For supply pressure ratios, P_{t1}/P_{to}, greater than 1.89, $M_3^* = 1.0$ for all conditions. This renders the concept of M_{2cr}^* meaningless and p_2 is not equal to p_3, specifically $p_2 < p_3$.

$$\frac{p_3}{P_{to}} = \left(\frac{p_3}{P_{t1}}\right)\left(\frac{P_{t1}}{P_{to}}\right)$$

$$\frac{p_3}{P_{t1}} = \frac{p_3}{P_{t3}} = \left[1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}\right]$$

Since $M_3^* = 1.0$,

$$\frac{p_3}{P_{to}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{P_{t1}}{P_{to}}\right) \qquad \text{(A-20)}$$

and

$$\frac{p_2}{P_{to}} = \left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right]^{\frac{\gamma}{\gamma-1}}$$

The basic condition for constant pressure mixing is:

$$p_2 = p_4 \qquad \text{(A-21)}$$

Therefore $\frac{p_2}{P_{to}} = \left(\frac{p_4}{P_{t4}}\right)\left(\frac{P_{t5}}{P_{to}}\right)$ since $P_{t4} = P_{t5}$

Substituting the isentropic relationships for p_2/P_{to} and P_{t5}/P_{to} ($p_5 = P_{to}$) and raising to the $\frac{\gamma-1}{\gamma}$ power.

$$\left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right] = \left(\frac{P_{t5}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\left[1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}\right]$$

Hence M_4^* can be determined from

$$M_4^* = \left\{ \frac{\gamma+1}{\gamma-1} \left[1 - \frac{\left[1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}\right]}{\left(\frac{P_{t5}}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}} \right] \right\}^{\frac{1}{2}} \tag{A-22}$$

Analyzing the Equations (A-18) and (A-22), it is seen that M_3^* and M_4^* are fixed when P_{t1}/P_{to}, P_{t5}/P_{to} and M_2^* are fixed. T_{t1}/T_{to} does not appear in either equation.

The three basic equations that have to be satisfied for constant pressure mixing ($p_2 = p_4$) are continuity, energy and momentum.

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \tag{A-23}$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \tag{A-24}$$

Momentum:

$$(\dot{m}_3 + \dot{m}_2)V_4 - (\dot{m}_3 V_3 + \dot{m}_2 V_2) = (p_3 - p_2)A_3 \tag{A-25}$$

It is noted that the resultant pressure forces in (A-25) are zero in the constant pressure case when $M_3^* \leq 1$ and $M_2^* \leq M_{2cr}^*$ since $p_2 = p_3$. When $M_3^* = 1$ and $M_2^* > M_{2cr}^*$ then $p_2 < p_3$. The continuity equation (A-23) can be rearranged as follows:

$$\frac{\dot{m}_2}{\dot{m}_3} + 1 = \frac{\dot{m}_4}{\dot{m}_3}$$

Let $\quad x = \dot{m}_2/\dot{m}_3$

So,

$$x + 1 = \frac{\rho_4}{\rho_3} \frac{A_4}{A_3} \frac{M_4^*}{M_3^*} \frac{a_4^*}{a_3^*}$$

$$\frac{A_4}{A_3} = (x+1) \frac{\rho_3}{\rho_4} \frac{M_3^*}{M_4^*} \frac{a_3^*}{a_4^*} \tag{A-26}$$

and from the isentropic relationship:

$$\frac{\rho}{\rho_t} = \left[1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_3}{\rho_4} = \frac{\rho_{t3}}{\rho_{t4}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}}\right\}^{\frac{1}{\gamma-1}} = \frac{\frac{P_{t3}}{T_{t3}}}{\frac{P_{t5}}{T_{t5}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}}\right\}^{\frac{1}{\gamma-1}}$$

$$\frac{\frac{P_{t3}}{T_{t3}}}{\frac{P_{t5}}{T_{t5}}} = \frac{P_{t1}}{P_{t5}} \frac{T_{t5}}{T_{t1}} = \frac{\frac{P_{t1}}{P_{t0}}}{\frac{P_{t5}}{P_{t0}}} \frac{\frac{T_{t5}}{T_{t0}}}{\frac{T_{t1}}{T_{t0}}}$$

Hence,

$$\frac{\rho_3}{\rho_4} = \frac{T_{t5}}{T_{t0}} \frac{\frac{P_{t1}}{P_{t0}}}{\frac{T_{t1}}{T_{t0}}} \frac{1}{\frac{P_{t5}}{P_{t0}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}}\right\}^{\frac{1}{\gamma-1}}$$

$$\frac{a_3^*}{a_4^*} = \left(\frac{T_{t3}}{T_{t4}}\right)^{\frac{1}{2}} = \left(\overset{1}{\cancel{\frac{T_{t3}}{T_{t1}}}} \frac{T_{t1}}{T_{t0}} \frac{T_{t0}}{T_{t4}}\right)^{\frac{1}{2}} = \left(\frac{T_{t1}}{T_{t0}} \frac{T_{t0}}{T_{t4}}\right)^{\frac{1}{2}}$$

Substituting for ρ_3/ρ_4 and a_3^*/a_4^* into Equation (A-26).

$$\frac{A_4}{A_3} = (\chi+1)\left(\frac{T_{t4}}{T_{t0}}\right)^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{t0}}}{\left(\frac{P_{t5}}{P_{t0}}\right)\left(\frac{T_{t1}}{T_{t0}}\right)^{\frac{1}{2}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}}\right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*}$$

It will be shown from the energy equation (A-31) that;

$$\frac{T_{t5}}{T_{t0}} = \frac{T_{t4}}{T_{t0}} = \frac{\left(\frac{T_{t1}}{T_{t0}} + \chi\right)}{1 + \chi}$$

Substituting for T_{t4}/T_{t0}, then A_4/A_3 becomes;

$$\frac{A_4}{A_3} = \left[(\chi+1)\left(\frac{T_{t1}}{T_{t0}} + \chi\right)\right]^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{t0}}}{\frac{P_{t5}}{P_{t0}}\left(\frac{T_{t1}}{T_{t0}}\right)^{\frac{1}{2}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}}\right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*} \qquad \text{(A-27)}$$

This is the dimensionless form of the continuity equation.

Similarly replacing M_4^* by M_5^* and noting that $T_{t5} = T_{t4}$ and $P_{t5} = P_{t4}$ then;

$$\frac{A_5}{A_3} = \left[(\chi + 1)\left(\frac{T_{t1}}{T_{to}} + \chi\right)\right]^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{to}}}{\frac{P_{t5}}{P_{to}}\left(\frac{T_{t1}}{T_{to}}\right)^{\frac{1}{2}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}\right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_5^*} \tag{A-28}$$

Similarly for A_6/A_3 replacing M_5^* by $M_6^* = 1$

$$\frac{A_6}{A_3} = \left[(\chi + 1)\left(\frac{T_{t1}}{T_{to}} + \chi\right)\right]^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{to}}}{\frac{P_{t5}}{P_{to}}\left(\frac{T_{t1}}{T_{to}}\right)^{\frac{1}{2}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{\frac{2}{\gamma+1}}\right\}^{\frac{1}{\gamma-1}} M_3^* \tag{A-29}$$

From the mass flow ratio χ, it is possible to determine A_2/A_3.

$$\frac{\dot{m}_2}{\dot{m}_3} = \chi = \frac{\rho_2 A_2 a_2^* M_2^*}{\rho_3 A_3 a_3^* M_3^*}$$

and:

$$\frac{A_2}{A_3} = \chi \frac{\frac{P_{t1}}{P_{to}}}{\left(\frac{T_{t1}}{T_{to}}\right)^{\frac{1}{2}}} \left\{\frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}}\right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_2^*} \tag{A-30}$$

Consider the energy equation, (A-24).

$$\dot{m}_2 c_p T_{t2} + \dot{m}_3 c_p T_{t3} = \dot{m}_4 c_p T_{t4}$$

$$\dot{m}_2 T_{t2} + \dot{m}_3 T_{t3} = \dot{m}_4 T_{t4}$$

$$\frac{\dot{m}_2}{\dot{m}_3} + \frac{T_{t1}}{T_{to}} = \left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right)\frac{T_{t4}}{T_{to}}$$

The non-dimensional form of energy equation is

$$\left(\frac{T_{t1}}{T_{to}} + \chi\right) = (1 + \chi)\frac{T_{t4}}{T_{to}} \tag{A-31}$$

Now consider the momentum equation (A-25). Substituting for the velocities and rearranging

$$\dot{m}_3 a_3^* M_3^* + \dot{m}_2 a_2^* M_2^* + (p_3 - p_2) A_3 = (\dot{m}_3 + \dot{m}_2) a_4^* M_4^*$$

Dividing both sides by $\dot{m}_3 a_o^*$.

$$\frac{a_3^*}{a_o^*} M_3^* + \left(\frac{\dot{m}_2}{\dot{m}_3}\right) \frac{a_2^*}{a_o^*} M_2^* + \frac{(p_3 - p_2) A_3}{a_o^* \dot{m}_3} = \left(1 + \frac{\dot{m}_2}{\dot{m}_3}\right) \frac{a_4^*}{a_o^*} M_4^* \qquad \text{(A-32)}$$

Let

$$D = \frac{(p_3 - p_2) A_3}{a_o^* \dot{m}_3}$$

D can be expressed as follows

$$D = \left(1 - \frac{p_2}{p_3}\right) \frac{p_3}{a_o^*} \frac{1}{\rho_3 V_3} = \left(1 - \frac{p_2}{p_3}\right) \frac{\rho_3 g R T_3}{a_o^*} \frac{1}{\rho_3 a_3^* M_3^*}$$

$$D = \left(1 - \frac{p_2}{p_3}\right) \frac{g R T_3}{a_o^*} \frac{\sqrt{T_{t_3}}}{a_3^* M_3^* \sqrt{T_{t_3}}}$$

$$D = \left(1 - \frac{p_2}{p_3}\right) \frac{\gamma + 1}{2\gamma} \sqrt{\frac{T_{t_1}}{T_{to}}} \frac{1}{M_3^*} \left[1 - \frac{\gamma - 1}{\gamma + 1} M_3^{*2}\right]$$

For $M_3^* \leq 1$ and $M_2^* \leq M_{2cr}^*$, $p_2 = p_3$; therefore $D = 0$.

For $M_3^* = 1$ and $M_2^* > M_{2cr}^*$, $p_2 \neq p_3$.

Therefore:

$$D = \left(1 - \frac{p_2}{p_3}\right) \frac{\gamma + 1}{2\gamma} \sqrt{\frac{T_{t_1}}{T_{to}}} \left[\frac{2}{\gamma + 1}\right] \neq 0$$

$$D = \frac{1}{\gamma} \sqrt{\frac{T_{t_1}}{T_{to}}} \left(1 - \frac{p_2}{p_3}\right)$$

$$D = \frac{1}{\gamma} \sqrt{\frac{T_{t_1}}{T_{to}}} \left(1 - \frac{\frac{p_2}{P_{to}}}{\frac{p_3}{P_{to}}}\right)$$

$$D = \frac{1}{\gamma}\sqrt{\frac{T_{t1}}{T_{to}}}\left(1 - \frac{\frac{p_2}{P_{to}}}{\left(\frac{p_3}{P_{t1}}\right)\left(\frac{P_{t1}}{P_{to}}\right)}\right)$$

$$D = \frac{1}{\gamma}\sqrt{\frac{T_{t1}}{T_{to}}}\left\{1 - \left(\frac{1 - \frac{\gamma-1}{\gamma+1}M_2^{*2}}{\frac{2}{\gamma+1}}\right)^{\frac{\gamma}{\gamma-1}}\frac{P_{to}}{P_{t1}}\right\}$$

Let

$$\xi = \frac{1}{\gamma}\left\{1 - \left(\frac{1 - \frac{\gamma-1}{\gamma+1}M_2^{*2}}{\frac{2}{\gamma+1}}\right)^{\frac{\gamma}{\gamma-1}}\frac{P_{to}}{P_{t_1}}\right\}$$

$$D = \sqrt{\frac{T_{t_1}}{T_{t_o}}}\ \xi$$

Substituting into Equation (A-32), the dimensionless momentum equation is

$$\sqrt{\frac{T_{t1}}{T_{to}}}M_3^* + \chi M_2^* + \sqrt{\frac{T_{t1}}{T_{to}}}\ \xi = (1+\chi)\sqrt{\frac{T_{t4}}{T_{to}}}M_4^* \qquad \text{(A-33)}$$

By fixing P_{t5}/P_{to} , P_{t1}/P_{to}, T_{t1}/T_{to} , and M_2^* , it is possible to obtain an explicit solution from the continuity, energy, and momentum equations since there remains only three unknowns χ , T_{t4}/T_{to}, and A_4/A_3 .

The energy and momentum equations are solved simultaneously for χ and T_{t4}/T_{t_o}. By eliminating T_{t4}/T_{to} between the two equations, the following quadratic is obtained.

$$(M_4^{*2} - M_2^{*2})\chi^2 + \left\{\left(\frac{T_{t1}}{T_{to}} + 1\right)M_4^{*2} - 2\sqrt{\frac{T_{t1}}{T_{to}}}M_2^*(M_3^* + \xi)\right\}\chi + \frac{T_{t1}}{T_{to}}\left\{M_4^{*2} - (M_3^* + \xi)^2\right\} = 0$$

This equation can be solved by using the quadratic formula

$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

$$a = M_4^{*2} - M_2^{*2}$$

$$b = \left(\frac{T_{t1}}{T_{to}} + 1\right) M_4^{*2} - 2\sqrt{\frac{T_{t1}}{T_{to}}}\, M_2^{*} (M_3^{*} + \xi)$$

$$c = \frac{T_{t1}}{T_{to}} \left\{ M_4^{*2} - (M_3^{*} + \xi)^2 \right\}$$

Once χ is determined, then T_{t4}/T_{to} can be determined from the energy equation and A_4/A_3 can be determined from the continuity equation. In addition, it is now possible to determine the area ratios A_2/A_3, A_5/A_3, and A_6/A_3 .

4. Development of constant area theory:

This development was provided by Dr. Gawain in Reference 2. Basically, it is a process to non-dimensionalize the equations of continuity, energy and momentum. These equations are initially in the following form:

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \qquad \text{(A-34)}$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \qquad \text{(A-35)}$$

Momentum:

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \qquad \text{(A-36)}$$

First, consider the continuity equation (A-34). Expanding this equation gives the relationship:

$$\rho_2 A_2 M_2^* a_2^* + \rho_3 A_3 M_3^* a_3^* = \rho_4 A_4 M_4^* a_4^*$$

It must be understood that $A_2 + A_3 = A_4$ for the constant area case.

The general relationship for the ratio of mass flow rate to the area is:

$$\dot{m}/A = \rho V = \rho a^* M^* = \frac{\rho}{\rho_t} \rho_t a^* M^* \tag{A-37}$$

Using the isentropic relationship for the density ratio and expanding $\rho_t a^*$ yields:

$$\dot{m}/A = \left\{1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right\}^{\frac{1}{\gamma-1}} \frac{P_t}{gRT_t} \left\{\frac{2\gamma}{\gamma+1} g R T_t\right\}^{1/2} M^*$$

Let

$$K_1 = \left\{\frac{2\gamma}{(\gamma+1) g R}\right\}^{1/2}.$$

The final form of Equation (A-37) is:

$$\dot{m}/A = K_1 P_t (T_t)^{-1/2} \left(1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{\frac{1}{\gamma-1}} M^*$$

It is convenient to define a velocity function $\phi(M^*)$:

$$\phi(M^*) \equiv \frac{\rho}{\rho_t} M^* = \left(1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{\frac{1}{\gamma-1}} M^*$$

Solving for the mass flow rate gives:

$$\dot{m} = K_1 A P_t (T_t)^{-1/2} \phi(M^*) \tag{A-38}$$

Now substituting Equation (A-38) into Equation (A-34).

$$K_1 A_2 P_{t2} (T_{t2})^{-1/2} \phi(M_2^*) + K_1 A_3 P_{t3} (T_{t3})^{-1/2} \phi(M_3^*) = K_1 A_4 P_{t4} (T_{t4})^{-1/2} \phi(M_4^*)$$

It can be seen from Figure 2 that the total state conditions at stations 1 and 3, and stations 0 and 2 are equal. The preceding equation is non-dimensionalized by dividing through by $K_1 A_3 P_{t_0} (T_{t_0})^{-1/2}$. This results in the final non-dimensional form of the continuity equation.

$$y\,\phi(M_2^*) + \frac{P_{t1}}{P_{to}}\sqrt{\frac{T_{to}}{T_{t1}}}\,\phi(M_3^*) = (1+y)\,\frac{P_{t4}}{P_{to}}\sqrt{\frac{T_{to}}{T_{t4}}}\,\phi(M_4^*) \qquad \text{(A-39)}$$

The energy equation (A-35) for the constant area jet pump can be expanded to yield the relationship:

$$\rho_2 A_2 V_2 c_p T_{t2} + \rho_3 A_3 V_3 c_p T_{t3} = \rho_4 A_4 V_4 c_p T_{t4} \qquad \text{(A-40)}$$

In general, the term $\rho V T_t$ can be expanded into the form:

$$\rho V T_t = \rho \frac{\rho}{\rho_t} a^* M^* T_t = T_t \rho_t a^* \phi(M^*)$$

and,

$$T_t \rho_t a^* = \left\{\frac{2\gamma}{(\gamma+1) R g}\right\}^{1/2} P_t (T_t)^{1/2} = K_1 P_t (T_t)^{1/2}$$

Therefore;

$$\rho V T_t = K_1 P_t (T_t)^{1/2} \phi(M^*)$$

Substituting this relationship into Equation (A-40) and non-dimensionalizing in the same manner as before results in the final form of the energy equation.

$$y\,\phi(M_2^*) + \frac{P_{t1}}{P_{to}}\sqrt{\frac{T_{t1}}{T_{to}}}\,\phi(M_3^*) = (1+y)\,\frac{P_{t4}}{P_{to}}\sqrt{\frac{T_{t4}}{T_{to}}}\,\phi(M_4^*) \qquad \text{(A-41)}$$

The specific heat at constant pressure (c_p) is assumed to be constant.

Finally, considering the momentum equation (assuming steady, frictionless flow and the velocities are uniform at stations 2, 3, and 4):

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \qquad \text{(A-36)}$$

Rearranging and substituting for $\dot{m}$ and A_4 gives;

$$A_2(p_2 + \rho_2 V_2^2) + A_3(p_3 + \rho_3 V_3^2) = (A_2 + A_3)(p_4 + \rho_4 V_4^2)$$

Now consider ($p + \rho V^2$) in general. This factor can be expanded to yield,

$$p + \rho V^2 = p + \frac{p}{Rg T} a^{*2} M^{*2} = p\left(1 + \frac{2\gamma}{\gamma+1}\frac{T_t}{T} M^{*2}\right)$$

where

$$\frac{T_t}{T} = \left(1 + \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{-1}$$

and

$$\frac{p}{P_t} = \left(1 + \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{\frac{\gamma}{\gamma-1}}$$

Therefore;

$$p + \rho V^2 = \frac{p}{P_t} P_t \left\{\frac{1 + M^{*2}}{1 + \frac{\gamma-1}{\gamma+1} M^{*2}}\right\} = P_t \left\{1 + \frac{\gamma-1}{\gamma+1} M^{*2}\right\}^{\frac{1}{\gamma-1}} \left\{1 + M^{*2}\right\}$$

It is helpful to define $\psi(M^*)$ as;

$$\psi(M^*) \equiv \left\{1 + \frac{\gamma-1}{\gamma+1} M^{*2}\right\}^{\frac{1}{\gamma-1}} \left\{1 + M^{*2}\right\}$$

So the final form of the term($p + \rho V^2$) is:

$$p + \rho V^2 = P_t \, \psi(M^*) \tag{A-42}$$

Substituting Equation (A-42) into Equation (A-36) and non-dimensionalizing yields the momentum equation:

$$y\,\psi(M_2^*) + \frac{P_{t1}}{P_{to}}\,\psi(M_3^*) = (1 + y)\frac{P_{t4}}{P_{to}}\,\psi(M_4^*) \tag{A-43}$$

From the continuity equation, the mass flow ratio is determined to be;

$$\dot{m}_2/\dot{m}_3 = \chi = y\,\frac{P_{to}}{P_{t1}}\sqrt{\frac{T_{t1}}{T_{to}}}\,\frac{\phi(M_2^*)}{\phi(M_3^*)} \tag{A-44}$$

And from the energy equation, the mixing tube discharge temperature ratio is:

$$\frac{T_{t4}}{T_{to}} = \frac{\chi + T_{t1}/T_{to}}{\chi + 1} \tag{A-45}$$

Dividing Equation (A-41) by Equation (A-43) results in a quadratic relationship in terms of M_4^* .

$$\frac{\phi(M_4^*)}{\psi(M_4^*)} = \sqrt{\frac{T_{to}}{T_{t4}}} \left\{ \frac{y\,\phi(M_2^*) + \frac{P_{t1}}{P_{to}}\sqrt{\frac{T_{t1}}{T_{to}}}\,\phi(M_3^*)}{y\,\psi(M_2^*) + \frac{P_{t1}}{P_{to}}\,\psi(M_3^*)} \right\} \tag{A-46}$$

where,

$$\frac{\phi(M_4^*)}{\psi(M_4^*)} = \frac{M_4^*}{1 + M_4^{*2}}$$

It can be shown that the right side of Equation (A-46) must be equal to or less than one-half. Let,

$$f(M_4^*) = \frac{M_4^*}{1 + M_4^{*2}} \tag{A-47}$$

Taking the derivative of this function with respect to M_4^* .

$$\frac{df(M_4^*)}{dM_4^*} = \frac{1 - M_4^{*2}}{(1 + M_4^{*2})^2}$$

Setting the derivative equal to zero and solving for M_4^* yields,

$$M_4^{*2} = 1$$

Choosing the positive root and substituting into Equation (A-47) gives the maximum value of the function $f(M_4^*)$.

$$f(M_4^*)_{max} = 1/2$$

Therefore;

$$f(M_4^*) \leq 1/2$$

This condition is a limit for the constant area case.

Solutions for optimum constant area jet pumps are contained in Appendix C. An iterative procedure was used to satisfy the boundary condition that P_{t4}/P_{to} must equal P_{t5}/P_{to}. This was accomplished by varying the area ratio A_2/A_3 until P_{t4}/P_{to} was equal to the specified value of P_{t5}/P_{to}. It is emphasized that each line of the computer results represents the optimum pump of a family of constant area jet pumps.

APPENDIX B

Itemized cost estimate

The following pages show the itemized cost estimate for various test facility configurations. The numbers for each item up to item 22 corresponds to the numbers shown on Figures 26 through 34. The labor estimate was made by Mr. Robert Besel, Laboratory Supervisor for the Aeronautical Engineering Department.

Material estimates were solicited from numerous companies. The code used for denoting the company is as follows:

Code	Company
MC	McMaster-Carr Supply Co.
DU	Ducommun Metals and Supply Co.
RE	Reliance Steel Co.
ME	Meriam Instrument Co.
PA	Pacific Metals Co.
RY	Ryerson Steel
GC	General Controls Corp.
USC	United Sensor and Control Corp.

The total cost estimates are included in the following itemized cost estimates.

ITEM	Units	Unit Price	Maximum Price	Minimum Price
1. Pressure tank: Stock No. - 437P611(MC)	1		149.60	149.60
2. Flange: ASA 150 Forged Steel Slip-on flange, NPS-4 in. (DU)	3	20.00	60.00	60.00
3. Pipe: ASTM A-53 SCH 40	9 ft	1.80/ft	18.20	
4 inch pipe (RE)	15 ft			36.20
4. Orifice plates and flanges: (ME)				
4 in. pipe-Series 30 flange	1		32.00	32.00
orifice plate	1		10.00	10.00
6 in. pipe-Series 30 flange	1		50.50	50.50
orifice plate	1		11.25	11.25
5. Elbow: 90° radius elbow for 4 in. pipe, No. 406 (DU)	1		19.50	19.50
6. Pipe: Type 304 SCH 40 Stainless steel pipe (PA)				
6 inch pipe	8 ft	39.44/ft	316.60	
5 inch pipe	10 ft	35.53/ft	355.98	
	2 ft	35.53/ft		71.06
4 inch pipe	8 ft	26.22/ft	210.27	
7. Flange: Type 316 MSS Stainless steel slip-on flanges (DU)				
NPS-4 inch	6	21.37	128.22	
NPS-5 inch	8	33.69	268.92	
	2	33.69		67.38
NPS-6 inch	6	36.97	221.82	

	ITEM	Units	Unit Price	Maximum Price	Minimum Price
8.	Nozzle: Type 304 Stainless				
	steel round bar, 6 in. dia.	144 lb	.87/lb	128.58	
	Spec. QQ-S-763B (PA)	46 lb	.87/lb		42.86
9.	Annulus - Hot rolled steel ring, O.D. 10" (RY)				
	I.D. 4 in-3 in. thick	1		63.08	63.08
	I.D. 5 in-2½ in. thick	1		45.06	
	I.D. 6 in-2in. thick	1		36.05	
10.	Backing Ring - ASA 150 lb forged steel blind flange NPS - 10 inches (DU)	1		58.00	
11.	Sheet: cold rolled 12 gage steel (48" x 120") (PA)	175 lb	.22/lb	39.20	39.20
12.	Flange: ASA 150 forged steel slip-on flange, (DU)				
	NPS - 6 in.	2	27.00	54.00	54.00
	NPS - 4 in.	4	20.00		80.00
13.	Pipe: ASTM-A-53, SCH 40, 6 inch steel pipe (RE)	12 ft	2.85/ft	36.20	36.20
14.	Flange: Type 316 MSS stainless steel slip-on	3	90.38	271.14	
	"special order" O.D. 16". NPS 4,5,6 (DU)	1			90.38
15.	Flange: ASA 150 forged steel slip-on reducing flange (DU)				
	4 x 11" O.D.	1		49.00	
	5 x 11" O.D.	1		49.00	
	6 x 11" O.D.	2	49.00	98.00	

	ITEM	Units	Unit Price	Maximum Price	Minimum Price
16.	Tubing: AISI MT-1015, 10 Gage dia. 2.375", length 24' (RE)	77 lb	.67/lb	52.59	52.59
17.	Turn Buckles: Drop forged Type 302 (DU)	6	1.10	6.60	6.60
18.	Channel: Aluminum 6063-T5 Die #2388, Length 48' (PA)	20 lb	1.66/lb	32.20	
19.	Bar: Aluminum 2024-T4 rectangular bar, ¼" x 2" x 36" (PA)	21.5 lb	2.08/lb	44.72	
20.	Casters: SB-4689 BASSICK (DU)	8	6.20	49.60	
21.	Mounting plates: Mixing tube actuator mounts (local)	48 8	1.00	48.00	 8.00
22.	Valve: Electrically actuated remote controlled butterfly valve (GC)	1		326.10	326.10
23.	Tubing: Copper tubing, O.D. ¼" 50' coil (DU)	4	6.06	24.24	
24.	Tubing: Imperial poly-flo tubing, Stock #66-P (DU)	200 ft	.09/ft	17.80	
25.	Wall Taps: static pressure mixing tube wall taps (DU)	72	.50	36.00	

	ITEM	Units	Unit Price	Maximum Price	Minimum Price
26.	Connectors: wall tap to copper tube connectors, No. 768 - FS (DU)	72	1.89	136.08	
27.	Connectors: copper tube to plastic tube connectors, No. 262-PC (DU)	72	0.26	18.73	
28.	Plugs: Dryseal threaded steel pressure plugs (DU)	50	.15	7.60	
29.	Probe: Kiel - temperature probe, Order # KT-14-C-ca-B (USC)	3	30.00	90.00	90.00
30.	Receiver: USC - 1396-6	2	380.00	760.00	
31.	Mechanical Cables: USC-1103-30	2	68.00	136.00	
32.	Collets: USC-1499-125	2	1.50	3.00	3.00
	Proposal For Remote Electrically Controlled System				
33.	Transmitter: USC-125I	1		310.00	
34.	Motor Box: USC-1462-B	1		675.00	
	Total Material Cost For Remote Electrical Control System			5555.42	

ITEM		Units	Unit Price	Maximum Price	Minimum Price
	Alternate Proposal For Remote Manual Controlled System				
35.	Transmitter: USC-1793 replaces items 33 and 34	1		220.00	
	Total Material Cost For Remote Manual Control			4787.42	
	Instrumentation Necessary For Minimum Cost System				
36.	Manual traverse unit: replaces items 30, 31, 33, 34, and 35 USC-1108-12	2	72.00		144.00
	Total Cost For Minimum Priced System				1601.50

Labor Estimate

		Man Hours
1.	Drill and tap flanges for primary system	7
2.	Install mass flow orifice plates and flanges	7
3.	Fabrication of nozzles	90
4.	Fabrication of mixing tube sections	108
5.	Fabrication of bell-mouth annulus	48
6.	Fabrication of secondary air plenum	32
7.	Drill and tap flanges for secondary system	2
8.	Install butterfly valve	4
9.	Fabricate mixing tube stanchions	6
10.	Fabricate handling cart	16
11.	Assemble complete system and hook-up instrumentation	42
	Total estimated man hours	362

APPENDIX C

Computer Results

This appendix contains computer solutions for a set of optimum constant area and constant pressure heated jet pumps for the conditions specified. Each line of data represents the parameters defining one optimum pump. The momentum coefficient is constant for each page of data.

Also included is a sample computer program and solutions for one jet pump with a fixed area ratio, $A_2/A_3 = 2.16$, and fixed supply conditions. Each line represents the theoretical performance for the conditions specified.

The symbols used in the computer programs are as follows:

PT5PO	=	the ratio of the demanded discharge total pressure to the ambient pressure.
PnPm	=	pressure ratio, station (n) to (m).
TnTm	=	temperature ratio, station (n) to (m).
AnAm	=	area ratio, station (n) to (m).
SACHn	=	velocity ratio (M*) at station (n).
TT5TO	=	ratio of the discharge total temperature to the ambient temperature.
WMWJ	=	mass flow ratio, secondary to primary.
G	=	γ , ratio of specific heats = 1.4.
R	=	rho, density.

```
..JDB0481F,SUMMERS AND RIDDER   TEN MIN MAX
      PROGRAM JETPUMP
C     CONSTANT PRESSURE HEATED JETPUMP
      READ 5,XPT5PO,XT1TO,XP1PO,XSACH2,GAMMA
    5 FORMAT(5F10.3)
      G=GAMMA
      CA=(G-1.)/(G+1.)
      CB=G/(G-1.)
      CD=(G-1.)/G
      CE=(G+1.)/(G-1.)
      CG=2./(G+1.)
      CH=2.*G/(G+1.)
      CJ=1./(G-1.)
      CK=1./G
      CN=(G+1.)/(2.*G)
      PT5PO=XPT5PO
      DO20 J=1,3
      XJ=J
      T1TO=XT1TO
      SACH5=(CE*(1.-(1./PT5PO)**CD))**.5
      A5A6=((CG/(1.-CA*(SACH5**2.)))**CJ)/SACH5
      CMA=PT5PO*SACH5
      PRINT 61
   61 FORMAT(1H1,///30X,54HCONSTANT PRESSURE HEATED JETPUMP, OPTIMUM SOL
     1UTIONS          ////)
      PRINT 60
   60 FORMAT(4X,5HPT5PO 5X,4HA5A6 4X,5HSACH5 6X, 3HCMA  /)
      PRINT 70,PT5PO,A5A6,SACH5,CMA
   70 FORMAT (4F9.3)
      PRINT 80
   80 FORMAT(/3X,5H P1PO 4X,6H SACH2 3X,6H SACH3 3X,6H SACH4 3X,
     15H A2A3 4X,5H A5A4 4X,5H A3A6 4X,5H A4A6 4X, 5H T1TO 4X,
     26H TT5TO 3X,5H WMWJ 4X, 5H  CPM 4X,5H  COM /)
      DO 30 K=1,5
      XK=K
      P1PO=XP1PO
      DO 40 M=1,5
      XM=M
      IF(P1PO-PT5PO) 40,40,16
   16 CONTINUE
      POP1=1./P1PO
      TOT1=1./T1TO
      PT4PO=PT5PO
      B=T1TO**.5
      SACH2=XSACH2
      WMWJO=0.0
      DO 50 N=1,750
      XN=N
      P2PO=(1.-CA*(SACH2**2.))**CB
      P4PO=P2PO
      P4PT4=P4PO/PT4PO
      SACH4=(CE*(1.-(P4PT4**CD)))**.5
      SACH3=(CE*(1.-((P2PO*POP1)**CD)))**.5
      P3PO=P2PO
      IF(SACH3-1.)10,11,11
   11 SACH3=1.0
      P3PO=P1PO*(CG**CB)
   10 P2P3=P2PO/P3PO
      R3R2=(P3PO/(P2PO**CK))/(T1TO*(1.-(CA*(SACH3**2.))))
      D=B*CN*(1.-P2P3)*(1.-(CA*(SACH3*SACH3)))/SACH3
      AA=(SACH4*SACH4)-(SACH2*SACH2)
      BB2=(SACH4*SACH4)*(B*B+1.)
      BB=BB2-(2.*B*SACH2*SACH3)-(2.*SACH2*D)
      CC2=(B*B)*((SACH4*SACH4)-(SACH3*SACH3))
      CC=CC2-(2.*B*SACH3*D)-(D*D)
      WMWJ=(-BB+((BB*BB)-(4.*AA*CC))**.5)/(2.*AA)
      IF (WMWJ - WMWJO) 15,26,26
   26 WMWJO=WMWJ
   50 SACH2=XSACH2+.001*XN
   15 TT4TO=(WMWJ+T1TO)/(WMWJ+1.)
      TT5TO=TT4TO
      A2A3=WMWJ*B*R3R2*SACH3/SACH2
      R4R2=(P2PO**CD)/(TT4TO*(1.-CA*(SACH4**2.)))
      A4X =SACH_*A2A3+R3R2*B*SACH3
      A4A3=A4X/(R4R2*(TT4TO**.5)*SACH4)
      A5A4=((P2PO**CK)*SACH4)/SACH5
      A6A4=(((1.-CA*(SACH4**2.))/CG)**CJ)*SACH4
      A2A6= A2A3/(A4A3*A6A4)
      A3A6= A2A6/A2A3
      A236= (A2A3+1.)*A3A6
      A4A6= 1./A6A4
```

```
      CMX = CA*(1.+WMWJ)*SACH5*TT5TO**.5
      CMP= CMX/((P1PO**CD)-1.)
      CPM = 1./CMP
      CMQ = CMX/(T1TO-(P1PO**CD))
      CQM = 1./CMQ
     0PRINT 90, P1PO,SACH2,SACH3,SACH4,A2A3,A5A4,A3A6,
     1A4A6,T1TO,TT5TO,WMWJ,CPM,CQM
   90 FORMAT(13F9.4)
   40 P1PO = XP1PO+.4*XM
      PRINT 63
   63 FORMAT(////)
   30 T1TO = XT1TO+1.*XK
   20 PT5PO =XPT5PO+.1*XJ
      PRINT 62
   62 FORMAT (1H1)
      END
      END

 1.10      1.0       1.2            0.251      1.4
```

CONSTANT PRESSURE HEATED JETPUMP, OPTIMUM SOLUTIONS

PT5PO	A5A6	SACH5	CMA
1.100	1.690	.401	.442

P1PO	SACH2	SACH3	SACH4	A2A3	A5A4	A3A6	A4A6	T1TO	TT5TO	WMWJ	CPM	CQM
1.2000	.9610	1.0000	1.0295	1.0012	1.6885	.5002	1.0010	1.0000	1.0000	.8328	.4360	-.4360
1.6000	.9650	1.0000	1.0331	5.0047	1.6880	.1667	1.0013	1.0000	1.0000	3.1233	.5209	-.5209
2.0000	.9660	1.0000	1.0341	9.0079	1.6879	.1000	1.0014	1.0000	1.0000	4.4977	.5954	-.5954
2.4000	.9660	1.0000	1.0341	13.0115	1.6879	.0715	1.0014	1.0000	1.0000	5.4139	.6622	-.6622
2.8000	.9660	1.0000	1.0341	17.0151	1.6879	.0556	1.0014	1.0000	1.0000	6.0684	.7231	-.7231
1.2000	.5390	.7618	.6663	.9364	1.4659	.5954	1.1531	2.0000	1.5293	.8895	.3420	6.0541
1.6000	.7820	1.0000	.8696	4.3903	1.6556	.1894	1.0209	2.0000	1.2147	3.6587	.4183	2.4924
2.0000	.8280	1.0000	.9101	8.0381	1.6738	.1117	1.0098	2.0000	1.1543	5.4813	.4700	1.6762
2.4000	.8460	1.0000	.9261	11.7441	1.6791	.0790	1.0066	2.0000	1.1295	6.7226	.5175	1.3034
2.8000	.8570	1.0000	.9359	15.4619	1.6819	.0610	1.0050	2.0000	1.1160	7.6170	.5615	1.0802
1.2000	.3860	.6678	.5533	.9360	1.2943	.6744	1.3060	3.0000	2.0585	.8895	.2948	10.7310
1.6000	.5960	1.0000	.7119	4.2089	1.5224	.2131	1.1103	3.0000	1.4276	3.6775	.3843	4.9639
2.0000	.6740	1.0000	.7767	7.4430	1.5889	.1260	1.0638	3.0000	1.3017	5.6287	.4328	3.5193
2.4000	.7090	1.0000	.8064	10.8044	1.6140	.0887	1.0472	3.0000	1.2498	7.0073	.4745	2.8646
2.8000	.7280	1.0000	.8228	14.2227	1.6263	.0683	1.0393	3.0000	1.2218	8.0179	.5128	2.4858
1.2000	.3170	.6324	.5089	.9347	1.2151	.7184	1.3910	4.0000	2.5878	.8894	.2629	14.4878
1.6000	.4920	.9827	.6299	4.2005	1.4154	.2295	1.1942	4.0000	1.6413	3.6776	.3584	7.1230
2.0000	.5720	1.0000	.6925	7.2198	1.4993	.1371	1.1274	4.0000	1.4503	5.6623	.4080	5.1801
2.4000	.6120	1.0000	.7250	10.3662	1.5371	.0967	1.0996	4.0000	1.3704	7.0984	.4480	4.2811
2.8000	.6350	1.0000	.7440	13.5812	1.5574	.0744	1.0853	4.0000	1.3273	8.1659	.4840	3.7617
1.2000	.2750	.6135	.4845	.9348	1.1692	.7466	1.4456	5.0000	3.1170	.8895	.2396	17.6807
1.6000	.4280	.9561	.5826	4.2021	1.3430	.2419	1.2586	5.0000	1.8551	3.6777	.3371	9.0457
2.0000	.5030	1.0000	.6383	7.1231	1.4275	.1457	1.1841	5.0000	1.5994	5.6738	.3878	6.6950
2.4000	.5430	1.0000	.6694	10.1453	1.4700	.1031	1.1498	5.0000	1.4915	7.1381	.4273	5.5874
2.8000	.5680	1.0000	.6893	13.2171	1.4953	.0794	1.1303	5.0000	1.4331	8.2368	.4623	4.9440

CONSTANT PRESSURE HEATEO JETPUMP, OPTIMUM SOLUTICNS

PT5PO	A5A6	SACH5	CMA
1.200	1.309	.552	.662

P1PO	SACH2	SACH3	SACH4	A2A3	A5A4	A3A6	A4A6	T1TO	TT5TO	WMWJ	CPM	CQM
1.6000	.9280	1.0000	1.0593	2.0080	1.3030	.3338	1.0042	1.0000	1.0000	1.2472	.6954	-.6954
2.0000	.9310	1.0000	1.0618	4.0147	1.3025	.2003	1.0046	1.0000	1.0000	1.9958	.7948	-.7948
2.4000	.9320	1.0000	1.0626	6.0214	1.3024	.1431	1.0047	1.0000	1.0000	2.4950	.8841	-.8841
2.8000	.9330	1.0000	1.0634	8.0276	1.3022	.1113	1.0048	1.0000	1.0000	2.8515	.9655	-.9655
1.6000	.7700	1.0000	.9313	1.7719	1.3011	.3627	1.0057	2.0000	1.4054	1.4666	.5344	3.1837
2.0000	.8060	1.0000	.9598	3.6125	1.3060	.2172	1.0019	2.0000	1.2908	2.4387	.6095	2.1734
2.4000	.8220	1.0000	.9726	5.4774	1.3073	.1545	1.0009	2.0000	1.2436	3.1044	.6751	1.7003
2.8000	.8310	1.0000	.9798	7.3519	1.3079	.1198	1.0005	2.0000	1.2181	3.5855	.7348	1.4135
1.6000	.5950	1.0000	.8004	1.6908	1.2458	.3903	1.0504	3.0000	1.8079	1.4756	.4694	6.0632
2.0000	.6650	1.0000	.8511	3.3450	1.2735	.2363	1.0275	3.0000	1.5699	2.5094	.5415	4.4036
2.4000	.6940	1.0000	.8728	5.0587	1.2830	.1683	1.0199	3.0000	1.4714	3.2423	.6004	3.6251
2.8000	.7110	1.0000	.8857	6.7912	1.2879	.1304	1.0160	3.0000	1.4183	3.7810	.6531	3.1659
1.6000	.4920	.9827	.7310	1.6854	1.1950	.4076	1.0950	4.0000	2.2118	1.4757	.4244	8.4345
2.0000	.5670	1.0000	.7808	3.2415	1.2329	.2501	1.0613	4.0000	1.8507	2.5263	.4964	6.3027
2.4000	.6020	1.0000	.8053	4.8549	1.2488	.1789	1.0478	4.0000	1.6997	3.2878	.5528	5.2821
2.8000	.6230	1.0000	.8204	6.4909	1.2577	.1388	1.0404	4.0000	1.6179	3.8549	.6022	4.6797
1.6000	.4280	.9561	.6917	1.6861	1.1599	.4198	1.1281	5.0000	2.6157	1.4757	.3903	10.4713
2.0000	.5010	1.0000	.7368	3.1890	1.1998	.2601	1.0906	5.0000	2.1324	2.5322	.4616	7.9697
2.4000	.5370	1.0000	.7604	4.7406	.2183	.1869	1.0740	5.0000	1.9285	3.3079	.5165	6.7530
2.8000	.5580	1.0000	.7747	6.3238	1.2287	.1453	1.0650	5.0000	1.8179	3.8907	.5639	6.0312

CONSTANT PRESSURE HEATEO JETPUMP, OPTIMUM SOLUTIONS

PT5PO	A5A6	SACH5	CMA
1.300	1.162	.658	.856

P1PO	SACH2	SACH3	SACH4	A2A3	A5A4	A3A6	A4A6	T1TO	TT5TO	WMWJ	CPM	CQM
1.6000	.8870	1.0000	1.0786	1.0099	1.1530	.5011	1.0074	1.0000	1.0000	.6215	.8079	-.8079
2.0000	.8960	1.0000	1.0854	2.3528	1.1514	.3008	1.0088	1.0000	1.0000	1.1610	.9237	-.9237
2.4000	.8990	1.0000	1.0877	3.6954	1.1509	.2149	1.0092	1.0000	1.0000	1.5208	1.0276	-1.0276
2.8000	.9010	1.0000	1.0893	5.0374	1.1505	.1671	1.0096	1.0000	1.0000	1.7778	1.1223	-1.1223
1.6000	.7550	1.0000	.9809	.8954	1.1610	.5276	1.0004	2.0000	1.5766	.7344	.6015	3.5839
2.0000	.7840	1.0000	1.0018	2.1318	1.1615	.3192	1.0000	2.0000	1.4127	1.4229	.6932	2.4719
2.4000	.7980	1.0000	1.0120	3.3846	1.1613	.2280	1.0002	2.0000	1.3452	1.8965	.7711	1.9421
2.8000	.8060	1.0000	1.0179	4.6445	1.1611	.1772	1.0004	2.0000	1.3087	2.2396	.8412	1.6183
1.6000	.5950	1.0000	.8728	.8478	1.1389	.5517	1.0199	3.0000	2.1495	.7399	.5135	6.6328
2.0000	.6540	1.0000	.9111	1.9760	1.1505	.3391	1.0096	3.0000	1.8106	1.4674	.6013	4.8894
2.4000	.6790	1.0000	.9279	3.1357	1.1543	.2432	1.0063	3.0000	1.6701	1.9847	.6716	4.0545
2.8000	.6940	1.0000	.9382	4.3068	1.1562	.1892	1.0046	3.0000	1.5942	2.3660	.7335	3.5558
1.6000	.4920	.9827	.8111	.8451	1.1116	.5660	1.0449	4.0000	2.7242	.7400	.4562	9.0655
2.0000	.5620	1.0000	.8523	1.9093	1.1310	.3528	1.0270	4.0000	2.2104	1.4786	.5417	6.8787
2.4000	.5920	1.0000	.8709	3.0094	1.1382	.2544	1.0205	4.0000	1.9951	2.0148	.6083	5.8130
2.8000	.6100	1.0000	.8824	4.1233	1.1422	.1984	1.0170	4.0000	1.8785	2.4148	.6661	5.1763
1.6000	.4280	.9561	.7767	.8455	1.0919	.5762	1.0638	5.0000	3.2989	.7400	.4145	11.1222
2.0000	.4970	1.0000	.8139	1.8789	1.1131	.3623	1.0435	5.0000	2.6112	1.4826	.4976	8.5906
2.4000	.5290	1.0000	.832-	2.9398	1.1222	.2625	1.0350	5.0000	2.3209	2.0284	.5615	7.3412
2.8000	.5480	1.0000	.8438	4.0168	1.1274	.2053	1.0303	5.0000	2.1632	2.4389	.6164	6.5921

```
      PROGRAM JETPUMP
C     CONSTANT AREA HEATED JETPUMP, ITERATIVE SOLUTION
      READ 5,XPT5P0,XT1T0,XP1P0,XSACH2,GAMMA
    5 FORMAT(5F10.0)
      G=GAMMA
      CA=(G-1.)/(G+1.)
      CB=G/(G-1.)
      CD=(G-1.)/G
      CE=(G+1.)/(G-1.)
      CG=2./(G+1.)
      CH=2.*G/(G+1.)
      CJ=1./(G-1.)
      CK=1./G
      CN=(G+1.)/(2.*G)
      PT5P0=XPT5P0
      DO 20 J=1,3
      XJ=J
      PR=PT5P0
      T1T0=XT1T0
      SACH5=(CE*(1.-(1./PT5P0)**CD))**.5
      A5A6=((CG/(1.-CA*(SACH5**2.)))**CJ)/SACH5
      CMA=PT5P0*SACH5
      PRINT 60
 6    FORMAT(1H1,//36X,47HCONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTION
     1S /)
      PRINT 61
 61   FORMAT (3X,5HPT5P0 4X,4HA5A6 4X,5HSACH5 4X, 3HCMA /)
      PRINT 7 ,PT5P0,      A5A6,SACH5,CMA
 70   FORMAT (4F8.3)
      PRINT 80
 80   FORMAT(/3X,
     1          4HP1P0 4X,5HSACH2 3X,5HSACH3 3X,5HSACH4 3X,
     24HA2A3 4X,5H A5A4 3X,5H A3A6 3X,5H P4P0 3X,5HPT4P0 3X,
     34HT1T0 4X,5HTT5T0 3X,4HWMWJ 4X,4H CPM 4X,4H CQM/)
      DO 3  K=1,5
      XK=K
      P1P0=XP1P
      FINC = .[illegible]
      DO 4  M=1,
      XM=M
      IF(P1P0-PT5P0)4 ,4 ,26
 26   CONTINUE
      P0P1=1./P1P[illegible]
      T T1=1./T1T[illegible]
      PT4P0=PT5P0
      B=T1T0**.5
      SACH2=XSACH2
      [illegible]
      [illegible] N=1,375
      [illegible]
      [illegible]
      [illegible]
      P0P2=(1.-CA*(SACH2**2.))**CB
      SACH3=(CE*(1.-((P2P0*P0P1)**CD)))**.5
      [illegible]
      IF(SACH3-1.)1 ,11,11
```

```
11  SACH3=1.0
    P3P0=P1P0*(CG**CJ)
1   P2P3=P2P0/P3P0
    P2R0=P2P0**CK
    T3T1=(1.-CA*SACH3*SACH3)
    R3P0=P3P0/(T1T0*T3T1)
13  CONTINUE
    WMWJ=A2A3*R2R0*SACH2/(R3P0*B*SACH3)
    TT4T0=(WMWJ+T1T0)/(WMWJ+1.0)
    D=B*CN*(1.-P2P3)*(1.-(CA*(SACH3*SACH3)))/SACH3
    XAL=(R2R0*A2A3*SACH2+R3R0*B*SACH3)*(TT4T0**.5)/(A2A3+1.)
    XBE=((A2A3+1.)*CN*B*(1.-CA*(SACH3**2.)))/(P3P0*SACH3)
    XEP=(WMWJ*SACH2+B*SACH3+D+P2P0*XBE)/XBE
    XLA=((WMWJ+1.0)*(TT4T0**.5))/XBE
    XRAD=XEP*XEP+(XAL*CA-XLA)*4.*XAL
    ARAD=XRAD
    IF(ABSF(ARAD)-.0005) 56,56,55
56  ARAD=0.0
55  CONTINUE
    IF (ARAD) 33,57,57
57  CONTINUE
    SACH4=(-XEP+(ARAD**.5))/(2.*(XAL*CA-XLA))
    IF(SACH4) 53,53,54
54  CONTINUE
    P4P0=XEP-SACH4*XLA
    PT4P0=P4P0/(1.-CA*SACH4*SACH4)**CB
15  GO TO (1,2,3),NUMB
 1  IF(PR-PT4P0)16,3,17
 2  IF(PR-PT4P0)18,3,17
16  A2A3M=A2A3
    PT4P0M=PT4P0
    A2A3=A2A3+1.0
    GO TO 13
17  NUMB=2
    A2A3M=A2A3
    PT4P0M=PT4P0
    A2A3=A2A3-.1
    GO TO 13
18  NUMB=3
    A2A3=(A2A3M*(PT4P0-PR)-A2A3*(PT4P0M-PR))/(PT4P0-PT4P0M)
    GO TO 13
 3  IF(WMWJ-WMWJO)43,76,76
76  WMWJO=WMWJ
53  CONTINUE
    SACH2T=SACH2
    SACH4T=SACH4
    A2A3T=A2A3
    P4P0T=P4P0
    PT4P0T=PT4P0
    WMWJT=WMWJO
    TT4T0T=TT4T0
50  SACH2=XSACH2+0.002*XN
33  A5A4=(P4P0T**CK)*SACH4T/SACH5
    A6A4=(((1.-CA*(SACH4T**2 ))/CG)**CJ)*SACH4T
    A4A3=A2A3T+1.
    A2A6=A2A3T/(A4A3*A6A4)
```

```
      A3A6=A2A5/A2A3T
      A236=(A2A3T+1.)*A3A5
      TT5TO=TT4TOT
      CMX=CA*(1.+.A*JT)*SACH5*TT5TO**.5
      CMP=CMX/((P1PO**CD)-1.)
      CPM=1./CMP
      CMO=CMX/(T1TO-(P1PO**CD))
      COM=1./CMO
      PRINT 9 ,P1PO,SACH2T,SACH3,SACH4T,A2A3T,A5A4,A3A6,
     1P4P T,PT4POT,T1TO,TT TO, JT,CP ,CO 1
 9    FORMAT(2H *,F6.3,13F8.3)
      GO TO 
 43   A5A4=(P4PO**CK)*SACH4/SACH5
      A6A4=(((1.-CA*(SACH4**2 ))/C3)**CJ)*SACH4
      A4A3=A2A3+1.
      A2A6=A2A3/(A4A3*A6A4)
      A3A6=A2A6/A2A3
      A236=(A2A3+1.)*A3A5
      A4A6=1./A6A4
      TT5TO=TT4TO
      CMX=CA*(1.+XMJ)*SACH5*TT5TO**.5
      CMP=CMX/((P1PO**CD)-1.)
      CPM=1./CMP
      CMO=CMX/(T1TO-(P1PO**CD))
      COM=1./CMO
      PRINT 91,P1PO,SACH2,SACH3,SACH4,A2A3,A5A4,A3A6,
     1P4PO,PT4PO,T1TO,TT5TO, J,CMP,COM
 91   FORMAT(2H  ,F6.3,13F8.3)
 99   IF(M-6)40,96,98
 98   IF(M-7)40,97,97
 96   FINC=.2
      GO TO 40
 97   FINC=.4
 40   P1PO=P1PO+FINC
      PRINT 92
 92   FORMAT (1H )
 3    T1TO=XT1TO+1.*XK
      PRINT 93
 93   FORMAT ( 3X,30HSTAR INDICATES SACH4 MAXIMUM
 2    PT5PO=XPT5PO+.1*XJ
      PRINT 62
 62   FORMAT (1H1)
      END
      END

1.10      1.0       1.2       0.252      1.4
```

CONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTIONS

PT5PO	A5A6	SACH5	CMA
1.100	1.690	.401	.442

P1PO	SACH2	SACH3	SACH4	A2A3	A5A4	A3A6	P4PO	PT4PO	T1TJ	TT5TJ	WMWJ	CPM	CWM
* 1.200	1.000	1.000	1.000	1.000	1.690	.500	.581	1.100	1.000	1.000	.833	.438	-.438
* 1.300	1.000	1.000	1.000	2.000	1.690	.333	.581	1.100	1.000	1.000	1.538	.458	-.458
* 1.400	1.000	1.000	1.000	3.000	1.690	.250	.581	1.100	1.000	1.000	2.143	.480	-.480
* 1.500	1.000	1.000	1.000	4.000	1.690	.200	.581	1.100	1.000	1.000	2.667	.501	-.501
* 1.600	1.000	1.000	1.000	5.000	1.690	.167	.581	1.100	1.000	1.000	3.125	.521	-.521
* 1.800	1.000	1.000	1.000	7.000	1.690	.125	.581	1.100	1.000	1.000	3.889	.559	-.559
* 2.000	1.000	1.000	1.000	9.000	1.690	.100	.581	1.100	1.000	1.000	4.500	.595	-.595
* 2.400	1.000	1.000	1.000	13.000	1.690	.071	.581	1.100	1.000	1.000	5.417	.662	-.662
* 2.800	1.000	1.000	1.000	17.000	1.690	.056	.581	1.100	1.000	1.000	6.071	.723	-.723
1.200	.538	.761	.665	.939	1.465	.595	.841	1.100	2.000	1.529	.891	.342	6.051
1.300	.632	.897	.741	1.826	1.554	.385	.786	1.100	2.000	1.372	1.689	.369	4.376
1.400	.712	1.000	.810	2.652	1.617	.286	.734	1.100	2.000	1.293	2.413	.389	3.463
1.500	.754	1.000	.845	3.510	1.641	.228	.706	1.100	2.000	1.246	3.069	.404	2.887
* 1.600	.760	1.000	.841	4.444	1.639	.189	.709	1.100	2.000	1.215	3.656	.419	2.494
* 1.800	.754	1.000	.820	6.366	1.624	.141	.726	1.100	2.000	1.177	4.638	.447	1.996
* 2.000	.750	1.000	.808	8.292	1.615	.113	.735	1.100	2.000	1.156	5.423	.474	1.690
* 2.400	.744	1.000	.792	12.153	1.602	.080	.747	1.100	2.000	1.132	6.598	.525	1.324
* 2.800	.740	1.000	.783	16.017	1.595	.062	.754	1.100	2.000	1.119	7.434	.573	1.102
1.200	.386	.668	.553	.937	1.294	.674	.916	1.100	3.000	2.058	.891	.295	10.726
1.300	.456	.791	.602	1.824	1.374	.436	.884	1.100	3.000	1.744	1.689	.328	8.090
1.400	.516	.890	.650	2.644	1.443	.321	.852	1.100	3.000	1.586	2.413	.351	6.614
1.500	.560	.967	.684	3.440	1.488	.256	.828	1.100	3.000	1.491	3.072	.369	5.641
1.600	.594	1.000	.710	4.219	1.520	.213	.809	1.100	3.000	1.428	3.678	.384	4.964
* 1.800	.536	1.000	.741	5.846	1.555	.159	.786	1.100	3.000	1.348	4.740	.410	4.074
* 2.000	.532	1.000	.723	7.715	1.535	.126	.799	1.100	3.000	1.303	5.607	.434	3.530
* 2.400	.624	1.000	.700	11.484	1.508	.090	.817	1.100	3.000	1.253	6.898	.480	2.900
* 2.800	.518	1.000	.685	15.265	1.490	.070	.827	1.100	3.000	1.227	7.810	.524	2.559
1.200	.316	.632	.508	.938	1.214	.718	.943	1.100	4.000	2.587	.891	.263	14.482
1.300	.374	.750	.544	1.826	1.279	.468	.921	1.100	4.000	2.116	1.689	.297	11.166
1.400	.424	.845	.581	2.645	1.340	.346	.898	1.100	4.000	1.879	2.413	.322	9.262
1.500	.462	.920	.609	3.435	1.384	.275	.880	1.100	4.000	1.737	3.072	.342	8.012
1.600	.490	.982	.628	4.214	1.412	.230	.867	1.100	4.000	1.641	3.678	.358	7.123
1.800	.538	1.000	.665	5.702	1.464	.172	.842	1.100	4.000	1.522	4.751	.385	5.934
* 2.000	.556	1.000	.672	7.364	1.473	.137	.837	1.100	4.000	1.451	5.659	.408	5.183
* 2.400	.548	1.000	.644	11.078	1.435	.098	.856	1.100	4.000	1.374	7.019	.452	4.318
* 2.800	.542	1.000	.627	14.807	1.411	.076	.868	1.100	4.000	1.334	7.976	.493	3.851
1.200	.274	.613	.484	.939	1.168	.747	.957	1.100	5.000	3.116	.891	.239	17.673
1.300	.326	.729	.514	1.821	1.225	.489	.939	1.100	5.000	2.488	1.689	.274	13.821
1.400	.370	.821	.545	2.636	1.279	.363	.921	1.100	5.000	2.172	2.412	.300	11.366
1.500	.402	.895	.566	3.433	1.315	.290	.908	1.100	5.000	1.982	3.072	.320	10.106
1.600	.428	.956	.583	4.202	1.343	.242	.897	1.100	5.000	1.855	3.678	.337	9.046
1.800	.474	1.000	.618	5.630	1.398	.182	.874	1.100	5.000	1.695	4.754	.365	7.615
2.000	.504	1.000	.640	7.112	1.430	.146	.859	1.100	5.000	1.599	5.674	.388	6.695
* 2.400	.494	1.000	.605	10.817	1.379	.104	.882	1.100	5.000	1.495	7.079	.430	5.622
* 2.800	.488	1.000	.587	14.517	1 349	.081	.895	1.100	5.000	1.441	8.065	.470	5.023

STAR INDICATES SACH4 MAXIMUM

CONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTIONS

PT5P0	A5A6	SACH5	CMA
1.200	1.309	.552	.062

P1P0	SACH2	SACH3	SACH4	A2A3	A3A4	A5A6	P4P0	PT4P0	T1T0	TT5T0	WMWJ	CPM	CWM
* 1.300	1.000	1.000	1.000	.500	1.309	.667	.634	1.200	1.000	1.000	.385	.611	-.611
* 1.400	1.000	1.000	1.000	1.000	1.309	.500	.634	1.200	1.000	1.000	.714	.640	-.640
* 1.500	1.000	1.000	1.000	1.500	1.309	.400	.634	1.200	1.000	1.000	1.000	.668	-.668
* 1.600	1.000	1.000	1.000	2.000	1.309	.333	.634	1.200	1.000	1.000	1.250	.694	-.694
* 1.800	1.000	1.000	1.000	3.000	1.309	.250	.634	1.200	1.000	1.000	1.667	.746	-.746
* 2.000	1.000	1.000	1.000	4.000	1.309	.200	.634	1.200	1.000	1.000	2.000	.794	-.794
* 2.400	1.000	1.000	1.000	6.000	1.309	.143	.634	1.200	1.000	1.000	2.500	.883	-.883
* 2.800	1.000	1.000	1.000	8.000	1.309	.111	.634	1.200	1.000	1.000	2.857	.964	-.964
1.300	.642	.905	.836	.455	1.266	.710	.778	1.200	2.000	1.702	.425	.455	5.595
1.400	.722	1.000	.901	.880	1.293	.538	.721	1.200	2.000	1.554	.806	.487	4.541
1.500	.748	1.000	.913	1.325	1.297	.434	.711	1.200	2.000	1.464	1.154	.512	3.659
* 1.600	.760	1.000	.913	1.783	1.296	.363	.711	1.200	2.000	1.405	1.467	.534	3.184
* 1.800	.754	1.000	.876	2.737	1.284	.273	.743	1.200	2.000	1.334	1.994	.575	2.569
* 2.000	.750	1.000	.856	3.697	1.276	.219	.761	1.200	2.000	1.293	2.418	.613	2.185
* 2.400	.744	1.000	.833	5.626	1.264	.156	.781	1.200	2.000	1.247	3.055	.683	1.719
* 2.800	.740	1.000	.819	7.560	1.257	.122	.792	1.200	2.000	1.222	3.509	.746	1.435
1.300	.464	.796	.716	.453	1.182	.762	.878	1.200	3.000	2.404	.425	.383	9.462
1.400	.500	.881	.729	.903	1.194	.576	.867	1.200	3.000	2.108	.806	.419	7.877
1.500	.550	.962	.764	1.310	1.221	.464	.838	1.200	3.000	1.928	1.155	.446	6.820
1.600	.588	1.000	.791	1.705	1.240	.390	.816	1.200	3.000	1.808	1.476	.469	6.063
1.800	.536	1.000	.828	2.514	1.262	.295	.785	1.200	3.000	1.658	2.039	.508	5.049
* 2.000	.532	1.000	.798	3.443	1.244	.237	.811	1.200	3.000	1.571	2.502	.542	4.411
* 2.400	.524	1.000	.761	5.323	1.219	.170	.841	1.200	3.000	1.476	3.197	.606	3.658
* 2.800	.518	1.000	.740	7.214	1.202	.133	.859	1.200	3.000	1.426	3.691	.664	3.116
1.300	.382	.754	.668	.452	1.137	.793	.915	1.200	4.000	3.106	.425	.337	12.655
1.400	.412	.839	.676	.903	1.145	.601	.909	1.200	4.000	2.661	.806	.372	10.700
1.500	.454	.916	.703	1.309	1.171	.484	.888	1.200	4.000	2.392	1.155	.401	9.584
1.600	.486	.980	.723	1.702	1.188	.408	.872	1.200	4.000	2.212	1.476	.424	8.435
1.800	.540	1.000	.765	2.447	1.221	.311	.838	1.200	4.000	1.985	2.045	.463	7.140
* 2.000	.556	1.000	.763	3.287	1.220	.250	.840	1.200	4.000	1.851	2.525	.496	6.304
* 2.400	.548	1.000	.719	5.138	1.185	.180	.876	1.200	4.000	1.705	3.255	.556	5.514
* 2.800	.542	1.000	.694	7.002	1.162	.141	.896	1.200	4.000	1.629	3.772	.611	4.745
1.300	.332	.732	.643	.451	1.119	.812	.935	1.200	5.000	3.808	.425	.304	15.540
1.400	.358	.816	.646	.905	1.114	.617	.932	1.200	5.000	3.215	.806	.339	13.093
1.500	.394	.891	.667	1.313	1.136	.498	.916	1.200	5.000	2.856	1.155	.367	11.575
1.600	.422	.954	.683	1.706	1.152	.420	.904	1.200	5.000	2.616	1.476	.390	10.471
1.800	.472	1.000	.719	2.431	1.185	.322	.875	1.200	5.000	2.313	2.046	.429	8.959
2.000	.498	1.000	.733	3.204	1.197	.260	.864	1.200	5.000	2.132	2.532	.462	7.970
* 2.400	.494	1.000	.690	5.017	1.159	.188	.898	1.200	5.000	1.934	3.284	.519	6.782
* 2.800	.488	1.000	.662	6.367	1.131	.147	.920	1.200	5.000	1.831	3.815	.571	6.104

STAR INDICATES SACH4 MAXIMUM

CONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTIONS

PT5PO	A5A6	SACH5	CMA
1.300	1.162	.658	.856

P1PO	SACH2	SACH3	SACH4	A2A3	A5A4	A3A6	P4PO	PT4PO	T1TO	TT5TO	WMWJ	CPM	CWM
* 1.400	1.000	1.000	1.000	.335	1.161	.749	.687	1.300	1.000	1.000	.259	.742	-.742
* 1.500	1.000	1.000	1.000	.668	1.161	.600	.687	1.300	1.000	1.000	.445	.775	-.775
* 1.600	1.000	1.000	1.000	1.000	1.162	.500	.687	1.300	1.000	1.000	.625	.806	-.806
* 1.800	1.000	1.000	1.000	1.668	1.161	.375	.687	1.300	1.000	1.000	.926	.865	-.865
* 2.000	1.000	1.000	1.000	2.334	1.161	.300	.687	1.300	1.000	1.000	1.167	.921	-.921
* 2.400	1.000	1.000	1.000	3.667	1.162	.214	.687	1.300	1.000	1.000	1.528	1.025	-1.025
* 2.800	1.000	1.000	1.000	5.000	1.162	.167	.687	1.300	1.000	1.000	1.786	1.119	-1.119
1.400	.686	.989	.921	.302	1.153	.774	.763	1.300	2.000	1.788	.269	.542	4.829
* 1.500	.738	1.000	.966	.594	1.160	.628	.719	1.300	2.000	1.661	.514	.574	4.098
* 1.600	.750	1.000	.964	.898	1.160	.528	.721	1.300	2.000	1.577	.734	.602	3.584
* 1.800	.754	1.000	.928	1.525	1.154	.399	.756	1.300	2.000	1.474	1.111	.650	2.906
* 2.000	.750	1.000	.899	2.163	1.147	.320	.784	1.300	2.000	1.414	1.415	.695	2.479
* 2.400	.744	1.000	.867	3.448	1.137	.230	.814	1.300	2.000	1.348	1.872	.777	1.957
* 2.800	.740	1.000	.850	4.737	1.130	.179	.830	1.300	2.000	1.315	2.199	.851	1.657
1.400	.490	.876	.800	.306	1.106	.804	.876	1.300	3.000	2.576	.269	.452	8.497
1.500	.570	.972	.867	.570	1.136	.651	.814	1.300	3.000	2.319	.516	.485	7.412
1.600	.596	1.000	.876	.848	1.140	.551	.805	1.300	3.000	2.149	.741	.513	6.629
1.800	.630	1.000	.893	1.410	1.145	.421	.789	1.300	3.000	1.936	1.136	.561	5.572
* 2.000	.632	1.000	.862	2.016	1.135	.339	.818	1.300	3.000	1.811	1.465	.602	4.893
* 2.400	.624	1.000	.813	3.266	1.113	.245	.864	1.300	3.000	1.675	1.962	.676	4.080
* 2.800	.618	1.000	.786	4.526	1.097	.192	.889	1.300	3.000	1.603	2.316	.743	3.600
1.400	.402	.835	.753	.308	1.077	.825	.918	1.300	4.000	3.363	.269	.395	11.551
1.500	.470	.924	.808	.569	1.110	.667	.869	1.300	4.000	2.979	.516	.428	10.023
1.600	.492	.983	.812	.847	1.112	.566	.865	1.300	4.000	2.723	.741	.456	9.061
1.800	.540	1.000	.846	1.364	1.128	.435	.834	1.300	4.000	2.402	1.140	.503	7.742
* 2.000	.556	1.000	.841	1.925	1.126	.353	.838	1.300	4.000	2.210	1.479	.542	6.878
* 2.400	.548	1.000	.781	3.154	1.095	.255	.893	1.300	4.000	2.001	1.998	.611	5.837
* 2.800	.542	1.000	.749	4.396	1.074	.201	.922	1.300	4.000	1.891	2.368	.673	5.231
1.400	.350	.813	.729	.308	1.060	.838	.939	1.300	5.000	4.151	.269	.356	13.741
1.500	.410	.898	.777	.567	1.092	.679	.896	1.300	5.000	3.639	.516	.387	12.221
1.600	.430	.957	.780	.844	1.094	.576	.894	1.300	5.000	3.297	.741	.414	11.117
1.800	.474	1.000	.807	1.352	1.109	.445	.870	1.300	5.000	2.868	1.141	.460	9.595
2.000	.500	1.000	.822	1.871	1.117	.362	.856	1.300	5.000	2.611	1.483	.498	8.589
* 2.400	.494	1.000	.760	3.380	1.081	.263	.912	1.300	5.000	2.326	2.016	.563	7.363
* 2.800	.488	1.000	.724	4.313	1.055	.207	.944	1.300	5.000	2.178	2.396	.622	6.655

STAR INDICATES SACH4 MAXIMUM

```
..JOB0481F,SUMMERS AND RIDDER TEN MIN MAX

      PROGRAM JETPUMP

C     CONSTANT AREA HEATED JETPUMP, VARIABLE BACK PRESSURE
      READ 5     ,XT1TO,XP1PO,XSACH2,GAMMA,A2A3
    5 FORMAT(5F10.3)
      G=GAMMA
      CA=(G-1.)/(G+1.)
      CB=G/(G-1.)
      CD=(G-1.)/G
      CE=(G+1.)/(G-1.)
      CG=2./(G+1.)
      CH=2.*G/(G+1.)
      CJ=1./(G-1.)
      CK=1./G
      CN=(G+1.)/(2.*G)
      P1PO=XP1PO
      DO 40 M=1,7
      XM=M
      T1TO=XT1TO
      DO 30 K=1,3
      XK=K
      PRINT 61
   61 FORMAT(1H1,///30X,58HFIXED AREA HEATED JET PUMP, VARIABLE DISCHARG
     1E PRESSURE              ////)
      PRINT 60
   60 FORMAT(3X,4HP1PO 4X,4HT1TO 4X, 4HA2A3
      PRINT 70, P1PO,T1TO,A2A3
   70 FORMAT (3F8.3)
      PRINT 80
   80 FORMAT(/3X,
     15HPT5PO 3X,5HSACH2 3X, 5HSACH3 3X, 5HSACH4 3X, 5HSACH5 3X,
     2          4HA3A5 4X, 4HA5A6 4X, 5HTT5TO 3X, 5HPT4PO 3X, 4HP4PO 4X
     34HWMWJ 4X, 4H CPM 4X, 4H CQM 4X, 4H CMA  /)
   26 CONTINUE
      POP1=1./P1PO
      TOT1=1./T1TO
      B=T1TO**.5
      SACH2=XSACH2
      DO 50 N=1,101
      XN=N
      P2PO=(1.-CA*(SACH2**2.))**CB
      SACH3=(CE*(1.-((P2PO*POP1)**CD)))**.5
      P3PO=P2PO
      IF(SACH3-1.)10,11,11
   11 SACH3=1.0
      P3PO=P1PO*(CG**CB)
   10 P2P3=P2PO/P3PO
      R2RO=P2PO**CK
      T3T1=(1.-CA*SACH3*SACH3)
      R3RO=P3PO/(T1TO*T3T1)
      WMWJ=A2A3*R2RO*SACH2/(R3RO*B*SACH3)
      TT4TO=(WMWJ+T1TO)/(WMWJ+1.0)
      O=B*CN*(1.-P2P3)*(1.-(CA*(SACH3*SACH3)))/SACH3
      XAL=(R2RO*A2A3*SACH2+R3RO*B*SACH3)*(TT4TO**.5)/(A2A3+1.)
      XBE=((A2A3+1.)*CN*B*(1.-CA*(SACH3**2.)))/(P3PO*SACH3)
      XEP=(WMWJ*SACH2+B*SACH3+O+P2PO*XBE)/XBE
      XLA=((WMWJ+1.0)*(TT4TO**.5))/XBE
      XRAO=XEP*XEP+(XAL*CA-XLA)*4.*XAL
      ARAO=XRAO
      IF(ABSF(ARAO)-.0005) 56,56,55
   56 ARAO=0.0
   55 CONTINUE
      IF (ARAO) 30,57,57
   57 CONTINUE
      SACH4=(-XEP+(ARAO**.5))/(2.*(XAL*CA-XLA))
      IF(SACH4) 53,53,54
   54 CONTINUE
      P4PO=XEP-SACH4*XLA
      PT4PO=P4PO/(1.-CA*SACH4*SACH4)**CB
      PT5PO=PT4PO
      SACH5=(CE*(1.-(1./PT5PO)**CD))**.5
      A5A6=((CG/(1.-CA*(SACH5**2.)))**CJ)/SACH5
      CMA=PT5PO*SACH5
   76 WMWJD=WMWJ
   53 CONTINUE
      SACH2T=SACH2
```

```
      SACH4T=SACH4
      A2A3T=A2A3
      P4POT=P4PO
      PT4POT=PT4PO
      WMWJT=WMWJO
      TT4TOT=TT4TO
   33 A5A4=(P4POT**CK)*SACH4T/SACH5
      A6A4=(((1.-CA*(SACH4T**2 1)/CG1**CJ1*SACH4T
      A4A3=A2A3T+1.
      A2A6=A2A3T/(A4A3*A6A4)
      A3A6=A2A6/A2A3T
      A236=(A2A3T+1.1*A3A6
      TT5TO=TT4TOT
      CMX=CA*(1.+WMWJT)*SACH5*TT5TO**.5
      CMP=CMX/((P1PO**CD1-1.1
      CPM=1./CMP
      CMQ=CMX/(T1TO-(P1PO**CD11
      CQM=1./CMQ
     0PRINT 90,PT5PO,SACH2T,SACH3,SACH4T,SACH5,A3A6,A5A6,TT5TO,PT4PO,
     1P4POT,WMWJT,CPM,CQM,CMA
   90 FORMAT (14F8.3)
   50 SACH2=XSACH2+0.01*XN
   30 T1TO=XT1TO+1.*XK
   40 P1PO=XP1PO+.2*XM
      END
      END

 1.0       1.2       0.0       1.4       2.16
```

FIXED AREA HEATED JET PUMP, VARIABLE DISCHARGE PRESSURE

P1P0	T1T0	A2A3
1.600	1.000	2.160

PT5P0	SACH2	SACH3	SACH4	SACH5	A3A6	A5A6	TT5T0	PT4P0	P4P0	WMWJ	CPM	CQM	CMA
1.272	.000	.868	.254	.632	.812	1.192	1.000	1.272	1.225	.000	1.365	-1.365	.804
1.270	.010	.868	.260	.630	.794	1.194	1.000	1.270	1.221	.022	1.340	-1.340	.800
1.268	.020	.869	.266	.628	.776	1.197	1.000	1.268	1.217	.043	1.317	-1.317	.796
1.266	.030	.869	.273	.626	.759	1.199	1.000	1.266	1.212	.065	1.294	-1.294	.792
1.264	.040	.869	.279	.624	.742	1.202	1.000	1.264	1.208	.087	1.272	-1.272	.789
1.262	.050	.870	.286	.622	.727	1.204	1.000	1.262	1.203	.109	1.251	-1.251	.785
1.260	.060	.870	.292	.620	.711	1.207	1.000	1.260	1.199	.130	1.231	-1.231	.781
1.259	.070	.871	.299	.618	.697	1.209	1.000	1.259	1.194	.152	1.212	-1.212	.778
1.257	.080	.872	.305	.616	.683	1.212	1.000	1.257	1.190	.173	1.193	-1.193	.774
1.255	.090	.872	.312	.614	.670	1.214	1.000	1.255	1.185	.195	1.175	-1.175	.771
1.253	.100	.873	.319	.612	.657	1.217	1.000	1.253	1.180	.216	1.158	-1.158	.767
1.251	.110	.874	.326	.610	.644	1.219	1.000	1.251	1.176	.238	1.142	-1.142	.764
1.250	.120	.876	.332	.608	.632	1.222	1.000	1.250	1.171	.259	1.126	-1.126	.760
1.248	.130	.877	.339	.607	.621	1.224	1.000	1.248	1.166	.280	1.110	-1.110	.757
1.246	.140	.878	.346	.605	.610	1.226	1.000	1.246	1.161	.301	1.096	-1.096	.754
1.245	.150	.880	.353	.603	.599	1.229	1.000	1.245	1.157	.322	1.081	-1.081	.751
1.243	.160	.881	.360	.602	.589	1.231	1.000	1.243	1.152	.343	1.068	-1.068	.748
1.242	.170	.883	.367	.600	.579	1.234	1.000	1.242	1.147	.364	1.054	-1.054	.745
1.240	.180	.884	.374	.598	.569	1.236	1.000	1.240	1.142	.384	1.041	-1.041	.742
1.239	.190	.886	.381	.597	.560	1.238	1.000	1.239	1.137	.405	1.029	-1.029	.739
1.237	.200	.888	.388	.595	.551	1.240	1.000	1.237	1.132	.425	1.017	-1.017	.736
1.236	.210	.890	.395	.593	.542	1.243	1.000	1.236	1.127	.445	1.005	-1.005	.733
1.234	.220	.892	.402	.592	.534	1.245	1.000	1.234	1.122	.466	.994	-.994	.730
1.233	.230	.895	.410	.590	.526	1.247	1.000	1.233	1.116	.486	.983	-.983	.728
1.232	.240	.897	.417	.589	.518	1.249	1.000	1.232	1.111	.505	.973	-.973	.725
1.230	.250	.899	.424	.587	.510	1.252	1.000	1.230	1.106	.525	.963	-.963	.723
1.229	.260	.902	.431	.586	.503	1.254	1.000	1.229	1.101	.545	.953	-.953	.720
1.228	.270	.904	.439	.584	.496	1.256	1.000	1.228	1.095	.564	.943	-.943	.718
1.226	.280	.907	.446	.583	.489	1.258	1.000	1.226	1.090	.583	.934	-.934	.715
1.225	.290	.910	.453	.582	.483	1.260	1.000	1.225	1.084	.602	.925	-.925	.713
1.224	.300	.912	.461	.580	.476	1.262	1.000	1.224	1.079	.621	.917	-.917	.710
1.223	.310	.915	.468	.579	.470	1.264	1.000	1.223	1.073	.640	.908	-.908	.708
1.222	.320	.918	.476	.578	.464	1.266	1.000	1.222	1.068	.658	.900	-.900	.706
1.221	.330	.922	.483	.576	.459	1.268	1.000	1.221	1.062	.676	.892	-.892	.704
1.220	.340	.925	.491	.575	.453	1.270	1.000	1.220	1.057	.694	.885	-.885	.701
1.218	.350	.928	.498	.574	.448	1.272	1.000	1.218	1.051	.712	.878	-.878	.699
1.217	.360	.931	.506	.573	.442	1.274	1.000	1.217	1.045	.730	.870	-.870	.697
1.216	.370	.935	.513	.571	.437	1.276	1.000	1.216	1.039	.748	.863	-.863	.695
1.215	.380	.938	.521	.570	.432	1.278	1.000	1.215	1.034	.765	.857	-.857	.693
1.214	.390	.942	.528	.569	.428	1.280	1.000	1.214	1.028	.782	.850	-.850	.691
1.213	.400	.945	.536	.568	.423	1.281	1.000	1.213	1.022	.799	.844	-.844	.689
1.213	.410	.949	.544	.567	.419	1.283	1.000	1.213	1.016	.816	.838	-.838	.687
1.212	.420	.953	.551	.566	.414	1.285	1.000	1.212	1.010	.832	.832	-.832	.686
1.211	.430	.957	.559	.565	.410	1.287	1.000	1.211	1.004	.849	.826	-.826	.684
1.210	.440	.961	.567	.564	.406	1.288	1.000	1.210	.998	.865	.820	-.820	.682
1.209	.450	.965	.574	.563	.402	1.290	1.000	1.209	.992	.881	.815	-.815	.680
1.208	.460	.969	.582	.562	.399	1.292	1.000	1.208	.986	.897	.810	-.810	.679
1.207	.470	.973	.590	.561	.395	1.293	1.000	1.207	.980	.912	.804	-.804	.677

1.206	.480	.977	.597	.560	.391	1.295	1.000	1.206	.973	.927	.799	-.799	.675
1.206	.490	.982	.605	.559	.388	1.297	1.000	1.206	.967	.943	.794	-.794	.674
1.205	.500	.986	.613	.558	.385	1.298	1.000	1.205	.961	.958	.790	-.790	.672
1.204	.510	.991	.620	.557	.382	1.300	1.000	1.204	.955	.972	.785	-.785	.671
1.203	.520	.995	.628	.556	.379	1.301	1.000	1.203	.948	.987	.781	-.781	.669
1.203	.530	1.000	.636	.555	.376	1.303	1.000	1.203	.942	1.001	.776	-.776	.668
1.202	.540	1.000	.644	.554	.373	1.304	1.000	1.202	.936	1.015	.772	-.772	.666
1.201	.550	1.000	.651	.553	.370	1.306	1.000	1.201	.929	1.029	.768	-.768	.665
1.201	.560	1.000	.659	.553	.367	1.307	1.000	1.201	.923	1.043	.764	-.764	.663
1.200	.570	1.000	.667	.552	.365	1.309	1.000	1.200	.917	1.056	.760	-.760	.662
1.199	.580	1.000	.674	.551	.362	1.310	1.000	1.199	.910	1.069	.756	-.756	.661
1.199	.590	1.000	.682	.550	.360	1.311	1.000	1.199	.904	1.082	.753	-.753	.660
1.198	.600	1.000	.690	.550	.358	1.312	1.000	1.198	.897	1.095	.749	-.749	.659
1.198	.610	1.000	.698	.549	.355	1.314	1.000	1.198	.891	1.107	.746	-.746	.657
1.197	.620	1.000	.705	.548	.353	1.315	1.000	1.197	.884	1.119	.742	-.742	.656
1.197	.630	1.000	.713	.548	.351	1.316	1.000	1.197	.878	1.131	.739	-.739	.655
1.196	.640	1.000	.721	.547	.349	1.317	1.000	1.196	.871	1.142	.736	-.736	.654
1.196	.650	1.000	.729	.546	.347	1.318	1.000	1.196	.864	1.153	.733	-.733	.653
1.195	.660	1.000	.737	.546	.345	1.319	1.000	1.195	.858	1.164	.730	-.730	.653
1.195	.670	1.000	.744	.545	.343	1.320	1.000	1.195	.851	1.175	.727	-.727	.652
1.194	.680	1.000	.752	.545	.342	1.321	1.000	1.194	.845	1.185	.724	-.724	.651
1.194	.690	1.000	.760	.544	.340	1.322	1.000	1.194	.838	1.195	.722	-.722	.650
1.194	.700	1.000	.768	.544	.338	1.322	1.000	1.194	.831	1.205	.719	-.719	.649
1.193	.710	1.000	.776	.544	.337	1.323	1.000	1.193	.825	1.214	.716	-.716	.649
1.193	.720	1.000	.783	.543	.335	1.324	1.000	1.193	.818	1.223	.714	-.714	.648
1.193	.730	1.000	.791	.543	.334	1.325	1.000	1.193	.811	1.232	.712	-.712	.647
1.192	.740	1.000	.799	.542	.333	1.325	1.000	1.192	.804	1.241	.710	-.710	.647
1.192	.750	1.000	.807	.542	.331	1.326	1.000	1.192	.798	1.249	.707	-.707	.646
1.192	.760	1.000	.815	.542	.330	1.326	1.000	1.192	.791	1.257	.705	-.705	.646
1.192	.770	1.000	.822	.542	.329	1.327	1.000	1.192	.784	1.264	.703	-.703	.645
1.191	.780	1.000	.830	.541	.328	1.327	1.000	1.191	.777	1.271	.701	-.701	.645
1.191	.790	1.000	.838	.541	.327	1.328	1.000	1.191	.771	1.278	.700	-.700	.644
1.191	.800	1.000	.846	.541	.326	1.328	1.000	1.191	.764	1.285	.698	-.698	.644
1.191	.810	1.000	.854	.541	.325	1.329	1.000	1.191	.757	1.291	.696	-.696	.644
1.191	.820	1.000	.861	.540	.324	1.329	1.000	1.191	.750	1.297	.695	-.695	.643
1.191	.830	1.000	.869	.540	.323	1.329	1.000	1.191	.743	1.303	.693	-.693	.643
1.191	.840	1.000	.877	.540	.322	1.330	1.000	1.191	.737	1.308	.692	-.692	.643
1.190	.850	1.000	.885	.540	.322	1.330	1.000	1.190	.730	1.313	.690	-.690	.643
1.190	.860	1.000	.892	.540	.321	1.330	1.000	1.190	.723	1.318	.689	-.689	.642
1.190	.870	1.000	.900	.540	.320	1.330	1.000	1.190	.716	1.323	.688	-.688	.642
1.190	.880	1.000	.908	.540	.320	1.330	1.000	1.190	.709	1.327	.687	-.687	.642
1.190	.890	1.000	.916	.539	.319	1.331	1.000	1.190	.703	1.330	.686	-.686	.642
1.190	.900	1.000	.923	.539	.319	1.331	1.000	1.190	.696	1.334	.685	-.685	.642
1.190	.910	1.000	.931	.539	.318	1.331	1.000	1.190	.689	1.337	.684	-.684	.642
1.190	.920	1.000	.939	.539	.318	1.331	1.000	1.190	.682	1.340	.683	-.683	.642
1.190	.930	1.000	.947	.539	.318	1.331	1.000	1.190	.676	1.342	.683	-.683	.642
1.190	.940	1.000	.954	.539	.317	1.331	1.000	1.190	.669	1.344	.682	-.682	.642
1.190	.950	1.000	.962	.539	.317	1.331	1.000	1.190	.662	1.346	.682	-.682	.642
1.190	.960	1.000	.970	.539	.317	1.331	1.000	1.190	.655	1.347	.681	-.681	.642
1.190	.970	1.000	.977	.539	.317	1.331	1.000	1.190	.649	1.349	.681	-.681	.642
1.190	.980	1.000	.985	.539	.317	1.331	1.000	1.190	.642	1.349	.681	-.681	.642
1.190	.990	1.000	1.000	.539	.316	1.331	1.000	1.190	.629	1.350	.681	-.681	.642
1.190	1.000	1.000	1.000	.539	.316	1.331	1.000	1.190	.629	1.350	.681	-.681	.642

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6.	LT Carl R. Summers, USN 1315 South Third Street Aberdeen, South Dakota 57401	1
7.	LT Wendell C. Ridder R. R. #1 Higginsville, Missouri 64037	1

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13. ABSTRACT

The jet pump is a device that compresses low-pressure fluid by means of a high energy jet. This device is simple, reliable and light weight, and is attractive for various uses including eventual application to boundary layer control for aircraft.

In this study the theoretical flow of a perfect gas through constant pressure and constant area jet pumps is predicted by analyzing the equations of continuity, energy and momentum. Of particular interest is the effect of heating the high energy jet. The parameters describing optimum heated jet pumps are determined. The ideal pumps presented herein represent upper performance limits for actual devices.

Also included is a complete description and design of a facility for testing heated jet pumps. Specific test configurations are analyzed, and performance curves are illustrated. These curves facilitate comparison between theory and experiment.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Jet						
Pump						
Heated						
Boundary Layer						
Flow						
Mixing						
Compressible						
Subsonic						
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